Math 1050 CBE 3 Review
CBE 3

- Covers lessons 11-16
  - Quadratic Functions
  - Polynomials and Their Graphs
  - Polynomial Division
  - Real Zeros of a Polynomial
  - Fundamental Theorem of Algebra
  - Rational Functions and Their Graphs
Quadratic Functions
Equations, Vertex, and Graphing
Quadratic Functions

• Equation in different forms:

\[ f(x) = A(x - h)^2 + k \]
\[ f(x) = A(x - a)(x - b) \]
\[ f(x) = ax^2 + bx + c \]

• Each form has different advantages
Problem 1

- Determine the exact coordinates for the vertex and the exact coordinates for the x-intercepts of the parabola defined by the function: \( f(x) = \frac{2}{3}x^2 + \frac{4}{3}x + \frac{5}{6} \).

- Vertex at:

- Solution: Vertex: \((-1, 1/6)\) No x-int.
Problem 2 – Part 1

• Determine an equation for the parabola that has a vertex at the point (3,4) and contains the point (5, -1)

• Solution: 
  \[ f(x) = -\frac{5}{4}(x - 3)^2 + 4 \]
Problem 2 – Part 2

• Determine the y-coordinate for the point where the parabola intersects the y-axis

• Solution: $-\frac{29}{4}$
Polynomials and their Graphs
Definition, Zeros, Multiplicity, and End Behavior
What is a Polynomial?

• Any function that can be written in the form:

\[ f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x + a_0 \]

where \( n \) is a whole number and \( a_n, a_{n-1}, a_{n-2}, \ldots, a_1, a_0 \) are all real numbers.

• Leading Term: \( a_n x^n \)  
  Leading Coefficient: \( a_n \)

• Constant: \( a_0 \)  
  Degree of the Polynomial: \( n \)

• Can be written in many different forms.
Zeros of a Polynomial

- If \( f(c) = 0 \) then \( c \) is said to be a zero of the function \( f \).
- If \((x-c)\) is a factor of \( f \) then \( c \) must be a zero of \( f \).
- And if \( c \) is a zero of \( f \) then \((x-c)\) must be a factor of \( f \).
Multiplicity

- If \((x - c)^n\) is a factor of \(f\) then \(c\) is a zero with multiplicity of \(n\).

- What does the graph of different multiplicities look like?
  - \(n = 1?\)
  - \(n = 2?\)
  - \(n = 3?\)
End Behavior

• What happens to your outputs as inputs gets larger and larger positively? Larger and larger negatively?

• To determine the right end behavior, determine if the leading term of the polynomial is positive or negative for large positive values of x.

• To determine the left end behavior, determine if the leading term of the polynomial is positive or negative for large negative values of x.
Problem 3

• Choose the function from the following whose graph is shown, assume A, B, C, D are all positive real numbers

\[ f(x) = A(x + B)(x + C)(x - D)^2 \]
\[ h(x) = -A(x + B)(x + C)(x - D)^2 \]
\[ g(x) = -A(x - B)(x - C)(x + D)^2 \]
\[ t(x) = -A(x + B)(x + C)(x - D)^3 \]
\[ r(x) = A(x - B)^2(x + C) \]

• Determine zeros (negative and positive)
• Determine multiplicity
• Determine end-behavior

• Solution: h(x)
Problem 4

- Choose the function from the following whose graph is shown, assume A, B, C, D are all positive real numbers

\[ f(x) = -A(x + B)^2(x - C)(x - D)^2 \]
\[ h(x) = -A(x + B)(x - C)(x - D) \]
\[ g(x) = A(x - B)^2(x + C)(x + D)^2 \]
\[ t(x) = A(x + B)^2(x - C)(x - D)^2 \]
\[ r(x) = -A(x - B)^3(x - C)(x - D) \]

- Determine zeros (negative and positive)
- Determine multiplicity
- Determine end-behavior

- Solution: t(x)
Polynomial Division

equation and fundamentals
Polynomial Division

• Long division, set up just like you were dividing two numbers

• Synthetic Division
  • Can only use when dividing by a zero. \((x - c)\)
  • Set up with zero and coefficients
    • Don’t forget to write the zero coefficients
Problem 5

• Use polynomial division to write the rational expression as a polynomial in descending powers plus a remainder

\[
\frac{2x^3 - 6x^2 + 1}{x^2 + 2}
\]

• Solution:

\[
2x - 6 - \frac{4x - 13}{x^2 + 2}
\]
Real Zeros of Polynomials
Theorems, Factoring, and Rational Zeros
Rational Zeros Theorem

• If the reduced rational number p/q is a zero of a polynomial, then p must be a factor of the constant term and q must be a factor of the leading coefficient.

• In other words, any rational zeros must be a factor of the constant term divided by a factor of the leading coefficient.
Problem 6

- Find all the rational zeros of the polynomial function

\[ f(x) = 2x^3 - x^2 - 2x + 1 \]

- Use rational zeros theorem to find potential rational zeros

- Test values

- After finding one zero you can factor, use synthetic division, or test more values to find the rest

- Solution: \( \{1, -1, 1/2\} \)
Fundamental Theorem of Algebra
Theorem and Application
Fundamental Theorem of Algebra

• If \( f(x) \) is a polynomial with real number coefficients and a leading term of \( a_n x^n \), \( f(x) \) can be written in the form

\[
f(x) = a_n (x - c_1)(x - c_2)\ldots(x - c_n)
\]

where \( c_1, c_2, \ldots, c_n \) are complex numbers not necessarily unique.

• Polynomials of degree \( n \) have precisely \( n \) factors of the form \( (x-c) \)

• The sum of all multiplicities of all the zeros of a polynomial is equal to the degree
Problem 7 – part 1

• Use the graph to determine how many rational zeros there are for the function:

\[ f(x) = x^5 - 2x^4 - 8x^3 + 16x^2 + 7x - 14 \]

Use rational zeros Theorem

see if it matches any zeros on the graph

• Solution: 3 rational zeros
Problem 7 – part 2

• Use the graph to determine how many irrational zeros there are for the function:

\[ f(x) = x^5 - 2x^4 - 8x^3 + 16x^2 + 7x - 14 \]

Any zeros on the graph that are not rational must be irrational.

• Solution: 2 irrational zeros
Problem 7 – part 3

- Use the parts before to determine how many imaginary zeros there are for the function: 
  \[ f(x) = x^5 - 2x^4 - 8x^3 + 16x^2 + 7x - 14 \]

- Any zeros not rational or irrational must be imaginary
- Remember the degree = sum of multiplicity of all zeros

- Solution: 0 imaginary zeros
Problem 8

• Find all the zeros of the polynomial: \( p(x) = x^4 - 4x^3 - 3x^2 - 8x - 10 \)

• Hint one of the zeros has a value of \( \sqrt{2}i \)

• Remember imaginary and irrational zeros always come in pairs

• Use polynomial Division

• Factor

• Solution: \( \{-\sqrt{2}i, \sqrt{2}i, 5, -1\} \)
Rational Function Graphs

Holes, Asymptotes, and Intercepts.
What is a Rational Function?

• Any function where the output can be defined by dividing a polynomial by a polynomial
Holes

• Rational functions have holes for any input that is a zero of both the numerator and the denominator.

• Find holes by finding common factors of top and bottom to find the x coordinate, and using the reduced form to find the y coordinate.
Vertical Asymptotes

• Find vertical asymptotes by finding the zeros of the denominator of the reduced rational function
End Behavior

• The end behavior is a horizontal asymptote if:
  • The degree in the numerator is less than the degree in the denominator
    • Then the horizontal asymptote is at y=0
  • The degree in the numerator is equal to the degree in the denominator
    • Then the horizontal asymptote is found by dividing the leading coefficients

• The end behavior is an oblique (slant) asymptote if:
  • The degree in the numerator is exactly one greater than the degree in the denominator
    • Find the slant asymptote by dividing the numerator by the denominator and ignoring the remainder
Zeros

• Rational Functions have zeros when the numerator has a zero, but the denominator does not have a zero
Problem 9

Does the graph of the following function contain any holes?
If so, what are the coordinates for the hole/s?

\[ f(x) = \frac{x^2 + x - 2}{x^2 - 3x + 2} \]

Factor
Find common factors
Reduce
Plug in x

Solution: Yes, (1, -3)
Problem 10 – Part 1

- Consider the graph of the following function
- Does the graph have any vertical asymptotes? Where?

\[ f(x) = \frac{5x^2 + 5x - 30}{2x^2 - 4x} \]

- Factor
- Reduce
- Find zeros of denominator

- Solution: Yes, x=0
Problem 10 – Part 2

• Consider the graph of the following function
• Does the graph have any horizontal asymptotes? Where?

\[ f(x) = \frac{5x^2 + 5x - 30}{2x^2 - 4x}. \]

• Solution: Yes, Y=5/2
Problem 10 – Part 3

• Consider the graph of the following function
• Where does the graph cross the x-axis?

\[ f(x) = \frac{5x^2 + 5x - 30}{2x^2 - 4x}. \]

• Find zeros of numerator but not denominator

• Solution: (-3,0)
Problem 10 – Part 4

• Consider the graph of the following function

\[ f(x) = \frac{5x^2 + 5x - 30}{2x^2 - 4x} \]

• Plot horizontal asymptotes, vertical asymptotes, and zeros.
Problem 11 – Part 1

• Consider the graph of the following function

• Does the graph have any vertical asymptotes? Where?

\[ f(x) = \frac{2x^3 + 4x^2 - 6x}{x^2 + x - 2}. \]

• Solution: Yes, \( x = -2 \)
Problem 11 – Part 2

• Consider the graph of the following function

• Does the graph have an oblique asymptote? Where?

\[ f(x) = \frac{2x^3 + 4x^2 - 6x}{x^2 + x - 2}. \]

• Look at degree of numerator and degree of denominator

• Divide numerator by denominator and ignore remainder

• Solution: Yes, \( y = 2x + 2 \)
Problem 11 – Part 3

• Consider the graph of the following function
• Where does the graph cross the x-axis?

\[ f(x) = \frac{2x^3 + 4x^2 - 6x}{x^2 + x - 2}. \]

• Solution: (0,0) (-3,0)
Problem 11 – Part 4

• Consider the graph of the following function \[ f(x) = \frac{2x^3 + 4x^2 - 6x}{x^2 + x - 2}. \]
• Plot slant asymptotes, vertical asymptotes, and zeros.
Other Resources

• Aggie Math Learning Center
  • Visit usu.edu/math/amlc for more videos, resources, tutoring times, and recitation leader office hours