

STAT 5200 Handout #10: Factorial Design (Ch. 8-9)

Example: An experiment is carried out to evaluate the effects of two factors on the energy expenditure of honey bees. One factor is the **Temperature** at which the bees are kept. This factor has 3 levels: 20° C, 30° C, and 40° C. The other factor is the amount of **Sucrose** as a percentage of dissolved solids. (Additional solids are added so the liquid solution has the same viscosity irrespective of the amount of sucrose dissolved in it.) This factor has 3 levels also: 20%, 40%, and 60%. **Three** honey bees are randomly assigned to each combination of Temperature and Sucrose. A single measurement of the **Energy** expenditure of each honey bee is made. This experiment is a 3×3 factorial design with 3 replicates per combination of factor levels.

Like blinding to remove confounding factor (viscosity)

*↑ #Temp ↑ #Sucrose ↑ #reps
= # of exp. units per factor level comb.*

```
/* Enter data */
data bees;
  input Temp Sucrose $ Rep Energy @@;
  label Temp = 'Temperature in Deg. Celsius';
  label Sucrose = 'Sucrose as Pct. of Dissolved Solids';
  label Energy = 'Energy Expended, in Joules/Second';
cards;
  20 20% 1 3.1    20 20% 2 3.7    20 20% 3 4.7
  20 40% 1 5.5    20 40% 2 6.7    20 40% 3 7.3
  20 60% 1 7.9    20 60% 2 9.2    20 60% 3 9.3
  30 20% 1 6.0    30 20% 2 6.9    30 20% 3 7.5
  30 40% 1 11.5    30 40% 2 12.9    30 40% 3 13.4
  30 60% 1 17.5    30 60% 2 15.8    30 60% 3 14.7
  40 20% 1 7.7    40 20% 2 8.3    40 20% 3 9.5
  40 40% 1 15.7    40 40% 2 14.3    40 40% 3 15.9
  40 60% 1 19.1    40 60% 2 18.0    40 60% 3 19.9
;
run;

/* Fit two-way ANOVA model (with interaction) */
proc glm data = bees plots=diagnostic;
  class Temp Sucrose;
  model Energy = Temp Sucrose Temp*Sucrose;
  lsmeans Temp Sucrose Temp*Sucrose /
           pdiff adjust=Tukey lines;
  title1 'Two-way ANOVA';
run;

proc glm data = bees;
  class Sucrose Temp;
  model Energy = Temp Sucrose Temp*Sucrose;
run;
```

Two-way ANOVA

The GLM Procedure

Class Level Information		
Class	Levels	Values
Temp	3	20 30 40
Sucrose	3	20% 40% 60%

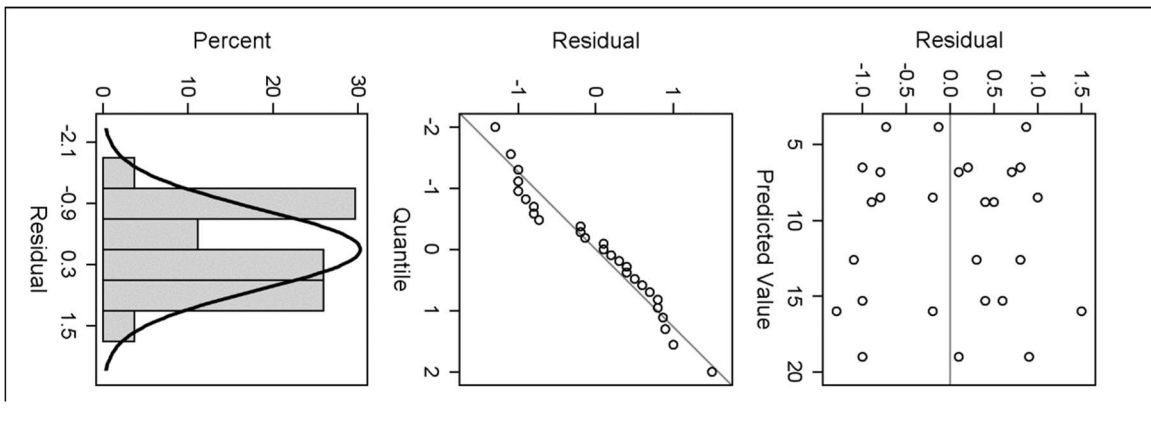
Dependent Variable: Energy Energy Expended, in Joules/Second

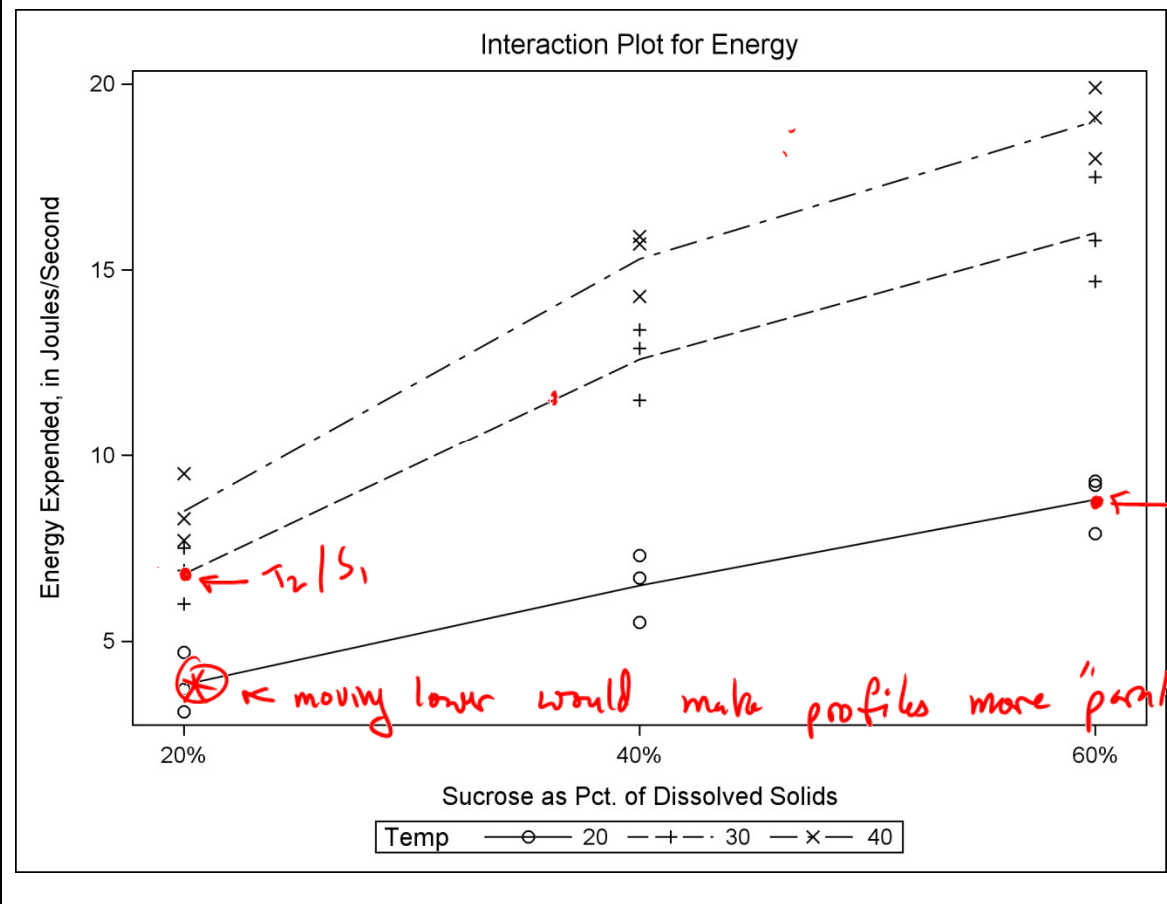
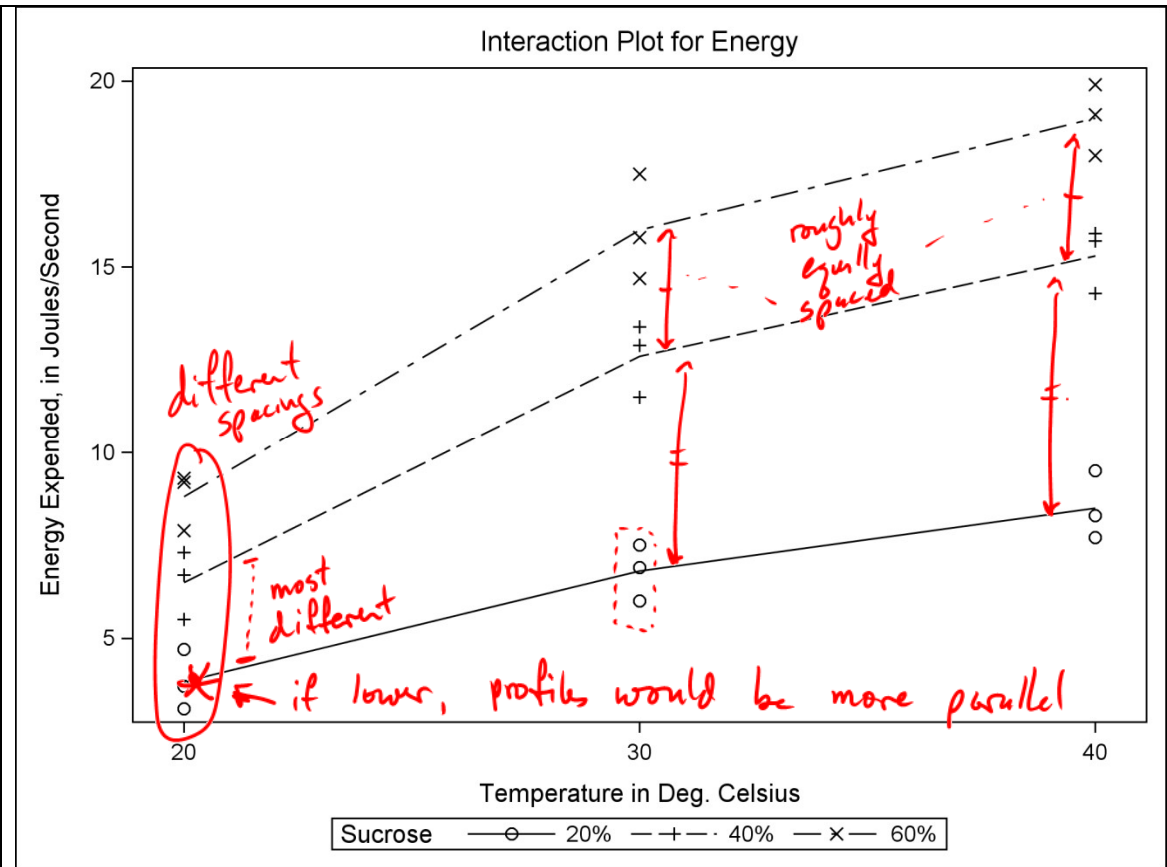
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	630.2474074	78.7809259	87.07	<.0001
Error	18	16.2866667	0.9048148		
Corrected Total	26	646.5340741			

R-Square	Coeff Var	Root MSE	Energy Mean
0.974809	8.795505	0.951218	10.81481

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Temp	3-1= ②	293.1585185	146.5792593	162.00	<.0001
Sucrose	3-1= ②	309.9585185	154.9792593	171.28	<.0001
Temp*Sucrose	4	27.1303704	6.7825926	7.50	0.0010

2 x 2





Tukey Comparison Lines for Least Squares Means of Temp			
LS-means with the same letter are not significantly different.			
	Energy LSMEAN	Temp	LSMEAN Number
A	14.266667	40	3
B	11.800000	30	2
C	6.377778	20	1

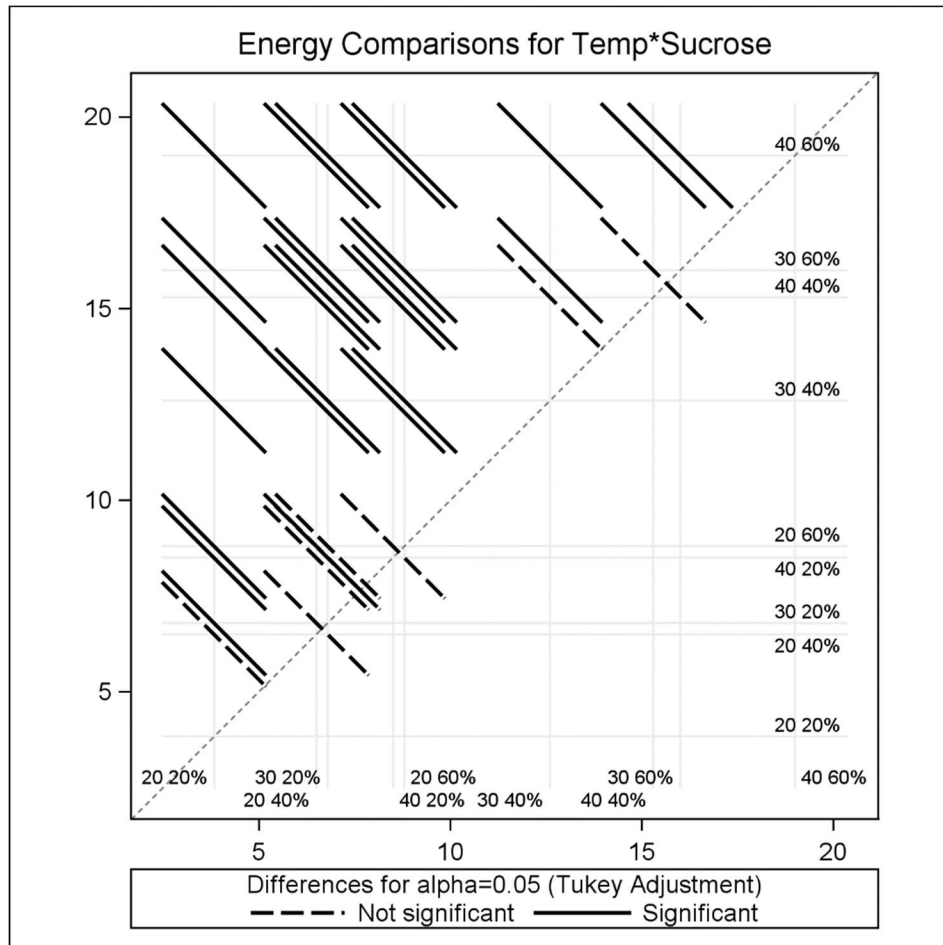
Tukey Comparison Lines for Least Squares Means of Sucrose			
LS-means with the same letter are not significantly different.			
	Energy LSMEAN	Sucrose	LSMEAN Number
A	14.600000	60%	3
B	11.466667	40%	2
C	6.377778	20%	1

Least Squares Means
Adjustment for Multiple Comparisons: Tukey

Temp	Sucrose	Energy LSMEAN	LSMEAN Number
20	20%	3.8333333	1
20	40%	6.5000000	2
20	60%	8.8000000	3
30	20%	6.8000000	4
30	40%	12.6000000	5
30	60%	16.0000000	6
40	20%	8.5000000	7
40	40%	15.3000000	8
40	60%	19.0000000	9

Least Squares Means for effect Temp*Sucrose
Pr > |t| for H0: LSMean(i)=LSMean(j)
Dependent Variable: Energy

i/j	1	2	3	4	5	6	7	8	9
1		0.0573	0.0001	0.0267	<.0001	<.0001	0.0003	<.0001	<.0001
2	0.0573		0.1378	1.0000	<.0001	<.0001	0.2621	<.0001	<.0001
3	0.0001	0.1378		0.2621	0.0029	<.0001	1.0000	<.0001	<.0001
4	0.0267	1.0000	0.2621		<.0001	<.0001	0.4511	<.0001	<.0001
5	<.0001	<.0001	0.0029	<.0001		0.0085	0.0013	0.0528	<.0001
6	<.0001	<.0001	<.0001	<.0001	0.0085		<.0001	0.9900	0.0245
7	0.0003	0.2621	1.0000	0.4511	0.0013	<.0001		<.0001	<.0001
8	<.0001	<.0001	<.0001	<.0001	0.0528	0.9900	<.0001		0.0038
9	<.0001	<.0001	<.0001	<.0001	<.0001	0.0245	<.0001	0.0038	



Tukey Comparison Lines for Least Squares Means of Temp*Sucrose					
LS-means with the same letter are not significantly different.					
		Energy LSMEAN	Temp	Sucrose	LSMEAN Number
	A	19.000000	40	60%	9
	B	16.000000	30	60%	6
	B				
C	B	15.300000	40	40%	8
C					
C		12.600000	30	40%	5
	D	8.800000	20	60%	3
	D				
	D	8.500000	40	20%	7
	D				
	D	6.800000	30	20%	4
	D				
E	D	6.500000	20	40%	2
E					
E		3.833333	20	20%	1

Interpretation: The significant interaction is “driven” by Temp=20 and Sucrose=20.

```

/* Fit same two-way ANOVA model, and look at some contrasts
-- both main effects and interaction,
but also to compare Temp20/Sucrose60
with Temp30/Sucrose20 */

```

```
proc glm data = bees;
```

```

class Temp Sucrose;

```

```
model Energy = Temp Sucrose Temp*Sucrose;
```

```
contrast 'Test interaction, by hand'
```

```

Temp*Sucrose 1 -1 0 /* This block: */
              -1 1 0 /* psi1 */
              0 0 0 ① → "and"

```

```

Temp*Sucrose 1 -1 0 /* This block: */
              0 0 0 /* psi2 */
              -1 1 0 ① → "and"

```

```

Temp*Sucrose 1 0 -1 /* This block: */
              -1 0 1 /* psi3 */
              0 0 0 ① → "and"

```

```

Temp*Sucrose 1 0 -1 /* This block: */
              0 0 0 /* psi4 */
              -1 0 1;

```

```
contrast 'psi 1 (10a p.3 top)'
```

```

Temp*Sucrose 1 -1 0 /* This block: */
              -1 1 0 /* T1-T2, S1-S2 */
              0 0 0;

```

```
contrast 'Temp20/Sucrose60 vs. Temp30/Sucrose20'
```

```

Temp 1 -1 0   Sucrose -1 0 1
Temp*Sucrose  0 0 1
              -1 0 0
              0 0 0;

```

```
title1 'Two-way ANOVA';
```

```
title2 '(and some contrasts)';
```

```
run;
```

**Two-way ANOVA
(and some contrasts)**

Dependent Variable: Energy Energy Expended, in Joules/Second

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	630.2474074	78.7809259	87.07	<.0001
Error	18	16.2866667	0.9048148		
Corrected Total	26	646.5340741			

R-Square	Coeff Var	Root MSE	Energy Mean
0.974809	8.795505	0.951218	10.81481

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Temp	2	293.1585185	146.5792593	162.00	<.0001
Sucrose	2	309.9585185	154.9792593	171.28	<.0001
Temp*Sucrose	4	27.1303704	6.7825926	7.50	0.0010

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Test interaction, by hand	4	27.1303704	6.7825926	7.50	0.0010
psi 1 (10a p.3 top)	1	7.3633333	7.3633333	8.14	0.0106
Temp20/Sucrose60 vs. Temp30/Sucrose20	1	6.0000000	6.0000000	6.63	0.0191

```

/* Check ave. of Temp20 over all Sucrose levels
vs. Temp30/Sucrose20 combination
(top of p. 4, Handout #10a).
Also check ave. of Temp40 over Sucrose20 and Sucrose40
vs. ave. of Temp30 over Sucrose40 and Sucrose60
(bottom of p. 4, Handout #10a)*/

proc glm data = bees;
  rw col
  class Temp Sucrose;

  model Energy = Temp Sucrose Temp*Sucrose;

  contrast '10a p.4 top'
    Temp 3 -3 0 Sucrose -2 1 1
    Temp*Sucrose 1 1 1 -3 0 0 0 0 0;

  contrast '10a p.4 bottom'
    Temp 0 -2 2 Sucrose 1 0 -1
    Temp*Sucrose 0 0 0 0 -1 -1 1 1 0;

  title1 'Check';

run;

```

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
10a p.4 top	1	0.40111111	0.40111111	0.44	0.5140
10a p.4 bottom	1	17.28000000	17.28000000	19.10	0.0004