STAT 5200 Handout #11a

Higher-Order and Unreplicated Factorial Designs (Ch. 8-9)

Higher-Order Factorial Designs: (like three-way design in Handout #11 example)

- Until now, we've considered factorial designs with [all combinations of] two factors
- Same ideas and methods apply when we have [all combinations of] more than two factors:
  - means model and effects model
  - constraints on parameters
  - model assumptions
  - contrasts (including post-hoc tests and multiple testing adjustments)
  - main effects and interactions
    - need to be careful interpreting higher-order interactions
  - interactions plots to help characterize significant interactions
    - even for higher-order interactions
  - partial R-square (p.4 of Ho # 8b)
  - ANOVA table
- Only change is interpretation of model terms

  → just a bit more complicated

Example: Handout #11 – Fabric data

- 3 factors – all combinations $3 \times 2 \times 2$

  A, B, C (Prop, Surf, Filler)

  i j k

- means model:

$$Y_{ijkl} = \mu_{ijk} + \epsilon_{ijkl}$$

$i = 1,2,3$
$j = 1,2$
$k = 1,2$
$l = 1,2$

- effects model:

$$Y_{ijkl} = \mu + \underbrace{A_i + B_j + C_k}_{\text{main effects}} + \underbrace{AB_{ij} + AC_{ik} + BC_{jk}}_{\text{2-way interactions}} + \underbrace{ABC_{ijk}}_{\text{3-way interactions}} + \underbrace{\epsilon_{ijkl}}_{\text{random error term}}$$

- constraints in effects model (here, sum to zero)
  - main effects, two-way interactions, and three-way interactions:

$$\sum_i A_i = \sum_j B_j = \sum_k C_k = 0$$

$$\sum_i AB_{ij} = \sum_j AB_{ij} = \sum_i AC_{ik} = \sum_k AC_{ik} = \sum_j BC_{jk} = \sum_k BC_{jk} = 0$$

$$\sum_i ABC_{ijk} = \sum_j ABC_{ijk} = \sum_k ABC_{ijk} = 0$$

  - These are called identifiability constraints, and ensure unique and interpretable parameter estimates [by OLS, minimizing $\sum_{ijkl}(\underbrace{Y_{ijkl} - \hat{Y}_{ijkl}}_{\text{residual}})^2$]

- In Handout #11, two pairwise interactions are significant

So all three factors involved in sig. interactions
Example: effect of Filler depends on level of Proportion
(interaction plot shows no Filler effect only when Prop = 25%,
or no real Prop. effect when Filler=2)

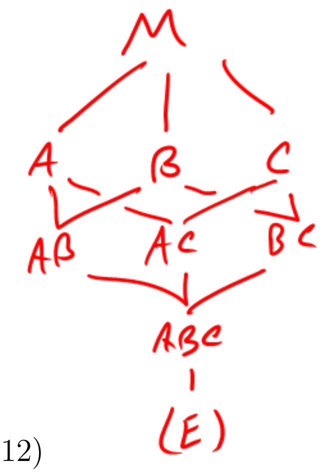
- In Handout #11, the three-way interaction is not significant

So "the effect of Filler depends on Proportion"
in essentially the same way for each level of Surface
→ See both int. plots for Surface=1 & Surface=2,
no Filler effect when Prop=25%,
(or no real Prop. effect when Filler=2)

- See Tukey groupings on last page of Handout #11

Probably best to use Surface=2 & Filler=2
(50% Prop. lowest, but not signif. different
from other Prop. levels in Tukey group "E")

- Recall Hasse diagram: (visualize what terms are in model)



Unreplicated Factorial Designs (like example in Handout #12)

- Lack of replicates causes problems in factorial design
  - Why a problem? No DF for error

    Sum of DF Type III table equals corrected total DF (N-1)

  - What to do?
    - Need to drop some parameters (set to zero)

    Usually, make assumption that at least one of the higher-order interaction terms is zero (drop it!)

- Example here (Handout #12)

$$Y_{ijkl} = \mu + A_i + B_j + C_k + AB_{ij} + AC_{ik} + BC_{jk} + \underbrace{ABC_{ijk} + \epsilon_{ijkl}}$$ $l=1$

ABC becomes the error term ← Combine or pool these terms; assume $ABC_{ijk} = 0$ $\forall$ ijk

- But – no real way to check this assumption

In HO #12, Soil × Salinity is the only sig. interaction. We can say that the effect of Soil (on EC) depends on Salinity in exactly the same way for all three levels of Water [We'd have to confirm the plausibility of this w/ a Content area specialist]

- Could set other interaction terms to zero

  ↳ like two-way, not just three-way int.

  - But no guarantee of which is correct to drop
  - Need to discuss implications very carefully w/ content area specialist

- Can do similar "pooling" of terms into error even with replicates
  - but see rules and warnings in section 8.10 of text