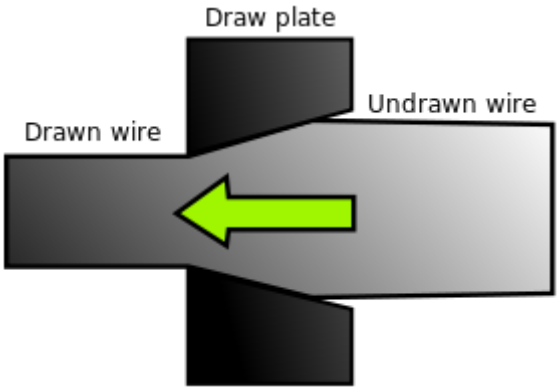
STAT 5200 Handout #13: One-Way Random Effects Design (Ch. 11)

Example: As part of a quality control exercise for a company, five drawing machines for making steel wire are randomly selected from a large number of machines, and then five samples of wire are run for each machine (for a total of 25 samples).



(http://en.wikipedia.org/wiki/File:Wiredrawing.svg)

For each sample a single measurement of the tensile strength of the wire is made. The purpose of the study is to determine what proportion of the variability in the measurements of tensile strength are due to differences among the machines.

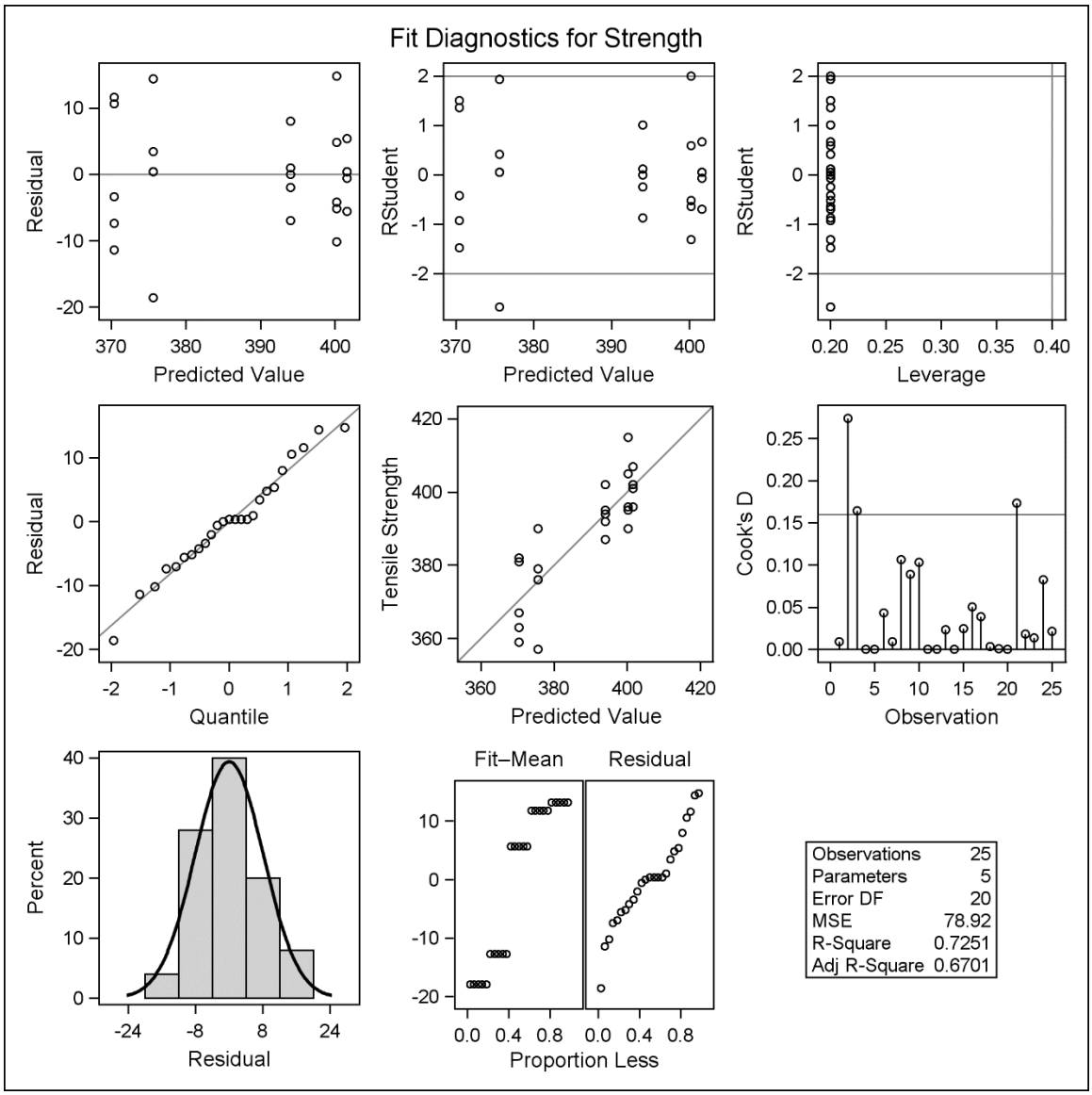
This steel wire example may be viewed as a completely randomized design with a random factor (machine).

```
/* Read in data */
data tensile;
  input Machine Strength @@;
  label Machine = 'Machine Number';
  label Strength = 'Tensile Strength';
  cards;
   1 379 1 357 1 390 1 376 1 376
   2 363 2 367 2 382 2 381 2 359
   3 401 3 402 3 407 3 402 3 396
   4 402 4 387 4 392 4 395 4 394
   5 415 5 405 5 396 5 390 5 395
   ;
run;
```

```
proc glm data=tensile plots=diagnostic;
  class Machine;
  model Strength = Machine;
  random Machine / test;
  title1 'PROC GLM with RANDOM statement';
run;
```

PROC GLM with RANDOM statement

The GLM Procedure



Dependent Variable: Strength Tensile Strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	4163.360000	1040.840000	13.19	<.0001
Error	20	1578.400000	78.920000		
Corrected Total	24	5741.760000			

MS_A / MS_E

R-Square	Coeff Var	Root MSE	Strength Mean
0.725102	2.287489	8.883693	388.3600

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Machine	4	4163.360000	1040.840000	13.19	<.0001

treats factor as fixed effect

Source	Type III Expected Mean Square
Machine	Var(Error) + 5 Var(Machine)

$\sigma^2 + 5\sigma_A^2$

Tests of Hypotheses for Random Model Analysis of Variance

Dependent Variable: Strength Tensile Strength

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Machine	4	4163.360000	1040.840000	13.19	<.0001
Error: MS(Error)	20	1578.400000	78.920000		

Treats factor as random effect

$MS_E = \hat{\sigma}^2$

"mixed" models have both fixed & random effects

```
proc mixed data=tensile covtest;
  class Machine;
  model Strength = ;
  random Machine;
  title1 'PROC MIXED with REML estimation (default)';
  title2 '-- this will avoid negative variance estimates';
run;
```

list fixed factors

list random factors

PROC MIXED with REML estimation (default)
-- this will avoid negative variance estimates

The Mixed Procedure

Model Information	
Data Set	WORK.TENSILE
Dependent Variable	Strength
Covariance Structure	Variance Components
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Containment

Convergence criteria met.

Covariance Parameter Estimates				
Cov Parm	**Estimate**	**Standard Error**	**Z Value**	**Pr > Z**
Machine	192.38	147.28	1.31	0.0957
Residual	78.9200	24.9567	3.16	0.0008

$\hat{\sigma}^2_A$

```
proc mixed data=tensile method=type3 covtest;
  class Machine;
  model Strength = ;
  random Machine;
  title1 'PROC MIXED with TYPE3 estimation';
  title2 '-- this mimics PROC GLM';
run;
```

PROC MIXED with TYPE3 estimation
-- this mimics PROC GLM

The Mixed Procedure

Model Information	
Data Set	WORK.TENSILE
Dependent Variable	Strength
Covariance Structure	Variance Components
Estimation Method	Type 3
Residual Variance Method	Factor
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Containment

Type 3 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	Expected Mean Square	Error Term	Error DF	F Value	Pr > F
Machine	4	4163.360000	1040.840000	Var(Residual) + 5 Var(Machine)	MS(Residual)	20	13.19	<.0001
Residual	20	1578.400000	78.920000	Var(Residual)	.	.	.	.

Covariance Parameter Estimates

Cov Parm	Estimate	Standard Error	Z Value	Pr Z
Machine	192.38	147.28	1.31	0.1915
Residual	78.9200	24.9567	3.16	0.0008