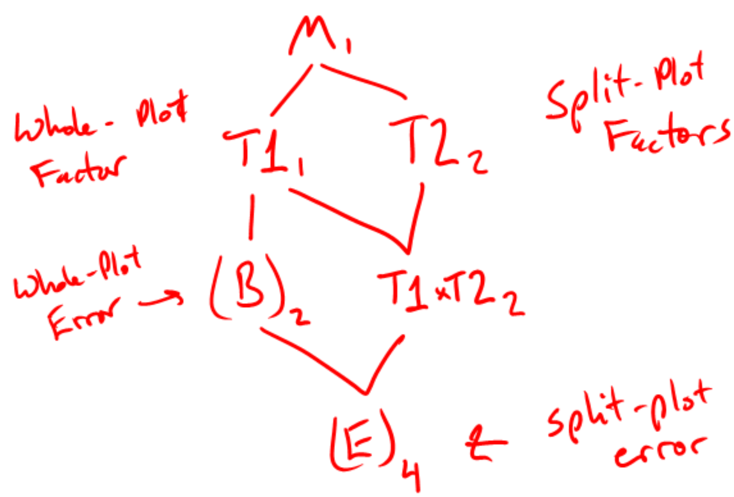
STAT 5200 Handout #20a Split-Plot Design

Classical Example – to explain "split-plot" name:

- Four plots of land (separate fields)
- Two treatment factors, with levels corresponding to amount or concentration:
  - Water; levels 1, 2 on smaller areas (harder to vary; think of one sprinkler line in field)
  - Fertilizer; levels 1, 2, 3 (easier to vary) on smaller areas

Field	Water	Fertilizer		
1	1	1	3	2
2	2	2	3	1
3	2	2	1	3
4	1	1	2	3

- Each whole plot is randomly assigned a Water level
- Each plot is split into three smaller regions that are each assigned a Fertilizer level — a separate randomization within each field
- Two types of experimental units (2 sizes): "Whole plot unit" (Field) and "Split plot unit" (Region)
- Here, two treatment factors (T1=Water — harder to vary, T2=Fertilizer — easier to vary) and one blocking factor (B=Field) — nested in T1
- Split-Plot Classical Example: Hasse Diagram and Model



$$Y_{ijk} = \mu + \underbrace{T1_i}_{\text{Water}} + \underbrace{B_{k(i)}}_{\text{Field}} + \underbrace{T2_j}_{\text{Fert.}} + T1 \times T2_{ij} + \epsilon_{k(ij)}$$

Key features of a Split-Plot design

- One blocking factor and two treatment factors
- One treatment factor is assigned to one larger size of unit [first randomization], called the whole-plot unit (usually this factor is harder to vary)
- Whole-plot units are split into smaller sized units, called split-plot units
- The second treatment factor is assigned to this smaller size of unit [second randomization]
- Must know how the two randomizations were done

  Just knowing # factor levels, or just having data, is not enough

See more complicated versions of split-plot designs on pp. 422-428:

- Whole plot treatment structure could actually be factorial combination of two or more other factors
- A block design could be used for the whole plots (Example 16.4)

Handout #20 Example 1: same design as textbook Example 16.4

- Whole-plot unit: field part

  randomly assigned to $\underbrace{\text{NSource}}_{\text{whole-plot factor}}$ levels in a block design

- Split-plot unit: field region

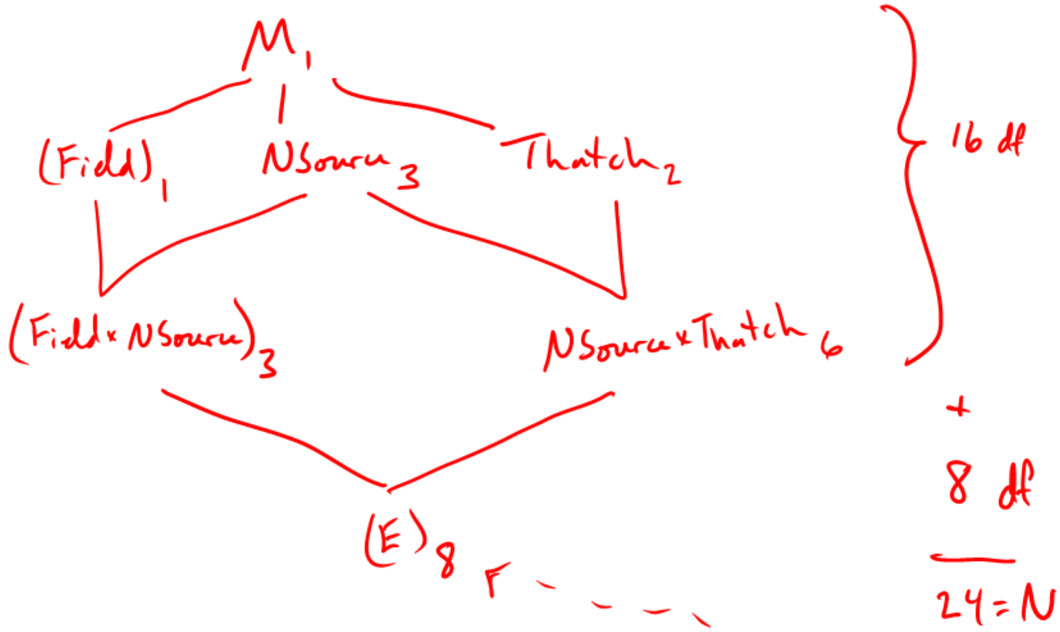
  randomly assigned to $\underbrace{\text{Thatch}}_{\text{split-plot factor}}$ levels within NSource×Field combo.

- Two randomizations (for two treatment factors), and two "sizes" of experimental units

  → Split plot design

Factors: Field, NSource, Thatch

- Hasse diagram:



Note: SPE term [E] is actually Field × Thatch (NSource) due to design's nested randomization; its DF account for this nesting:

$$DF_{Field} \times DF_{Thatch} \times (\#\text{ levels of } NSource) = 1 \times 2 \times 4 = 8$$

- Model:

$$Y_{ijkl} = \mu + \underset{i=1,\ldots,4}{NSource_i} + \underset{k=1,2}{\overset{\text{Block}}{Field_k}} + \overset{\text{Field} \times \text{NSource}}{Rep_{l(ik)}}$$

$$+ \underset{j=1\ldots3}{Thatch_j} + NSource \times Thatch_{ij} + \epsilon_{l(ijk)}$$

Note: l here indexes both:

- \# whole plot units receiving $NSource$ level i in $Field$ k
- \# split plot units receiving $Thatch$ level j in each $NSource_i \times Field_k$ combination

$l \equiv 1$