STAT 5200 Handout #8b

Tools for Factorial Design (Ch. 8)

$$
\begin{gathered}
Y_{ijk} = \mu + A_i + B_j + AB_{ij} + \epsilon_{ijk} \\
i = 1, \ldots, a; \quad j = 1, \ldots, b; \quad k = 1, \ldots, n; \quad \epsilon_{ijk} \text{ iid } N(0, \sigma^2) \\
\sum_i A_i = \sum_j B_j = \sum_i AB_{ij} = \sum_j AB_{ij} = 0
\end{gathered}
$$

General strategy in factorial design

1. Fit model (with A, B, and AB), and check assumptions

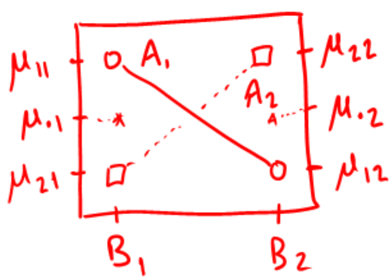
   ↳ transform Y if necessary
   ↳ Box-Cox

   ↳ constant variance & normality (& independence if relevant)

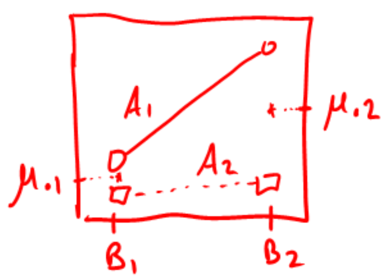
2. If AB test significant:

   (a) Main effects tests may be irrelevant or unrevealing, so ignore them (Real story is the interaction)

   - Consider two interaction plots to see why:



$\mu_{\cdot 1} = \mu_{\cdot 2}$ (no diff. in B levels)
But A level does affect the B_1 vs B_2 difference
Here, main effect test for B unrevealing (& not signif.)



Here, the test for B main effect ($\mu_{\cdot 1}$ vs $\mu_{\cdot 2}$) will be misleading (i.e., won't say what the effect of B is) because it doesn't look at A

   (b) Look at interaction plots and do posthoc mean comparisons (on AB) to characterize

   - look at what is "driving" the interaction (how does the effect of A depend on B - or vice-versa)

   ↳ note order in CLASS statement affects horiz. axis variable in interaction plots

3. If AB test not significant:

   (a) Look at main effect tests (A, B)

   (b) If A or B test significant, do posthoc tests to characterize

   ↳ use LSMEANS w/ ADJUST=TUKEY

4. Look at specific contrasts of interest

LSMEANS

- For unbalanced designs with more than one factor, the MEANS statement (and posthoc comparison options) are not appropriate
  ↳ incl. REGWQ
- Need to also account for: other factor effects
- LSMEANS (and posthoc comparison options) account for imbalance and for other effects
  use LSMEANS in multi-factor models
- REGWQ not defined here, so use Tukey's HSD
  - PDIFF (option in LSMEANS statement) with ADJUST=TUKEY
  - Simple summary: LINES option (see Handouts #8 (pp. 5 & 9) and #9)

Contrasts from one-way "combination" model (see p. 10 of Handout #8)

$$
\begin{aligned}
\text{effects model} \rightarrow \quad Y_{ijk} &= \mu + Glass_i + Phosphor_j + GlassPhosphor_{ij} + \epsilon_{ijk} \qquad i=1,2;\ j=1,2,3;\ k=1,2,3 \\
\text{means model} \rightarrow \quad &= \mu_l^{(c)} + \epsilon_{lk}, \qquad l = 1, \ldots, ab \qquad l=1,\ldots,6
\end{aligned}
$$

Compare Phosphor level A to Phosphor level C using $\mu_l^{(c)}$'s:

[words] H_0: (mean when Phosphor = A) = (mean when Phosphor = C)

$\equiv$

[parameters] $H_0: (\mu_1^{(c)} + \mu_4^{(c)})/2 = (\mu_3^{(c)} + \mu_6^{(c)})/2$

$\equiv$

[$\sim = 0$] $H_0: \underbrace{\mu_1^{(c)} - \mu_3^{(c)} + \mu_4^{(c)} - \mu_6^{(c)}} = 0$

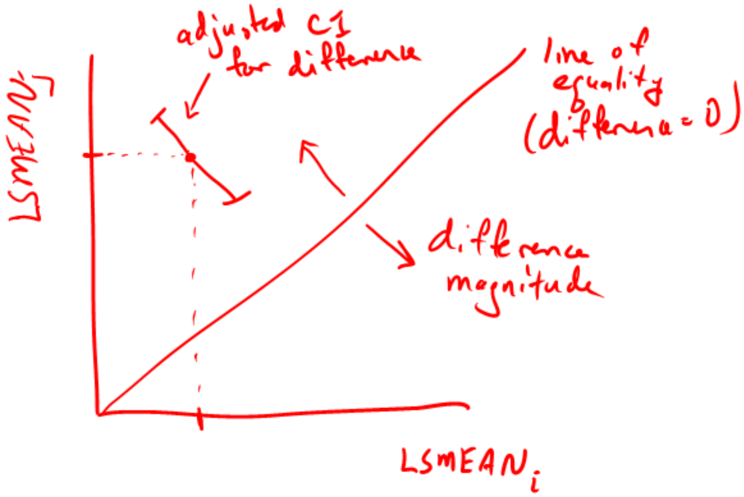
This is a contrast: $\psi = \mu_1^{(c)} + 0 \cdot \mu_2^{(c)} - \mu_3^{(c)} + \mu_4^{(c)} + 0 \cdot \mu_5^{(c)} - \mu_6^{(c)}$

1. linear combination of parameters
   general form is $\sum_l w_l \mu_l$
2. sum of coefficients is zero
   general form is $\sum_l w_l = 0$

Misc. Topics

- Diffogram: LSMEANS default summary (visualization), also called "mean=mean scatter plot" (Hsu 1996)

  Significant differences indicated by solid CI lines



- Partial R^2
  - Recall overall model $R^2 = SS_{Trt}/SS_{Total}$ = proportion of variation in Y explained by (combination of all) factor levels
  - For balanced designs, we can partition this to see contributions of each factor:

$$R^2_{partial} = SS_{effect} \;/\; SS_{total}$$

  - Example (Handout #8):

Effect	$R^2_{partial}$
Glass	$\frac{13{,}338.89}{15{,}461.11} = 0.863$
Phosphor	0.080
GlassPhosphor	0.003
Total:	$R^2 = 0.946$

    - So changes in Glass level, averaged equally over all levels of Phosphor, account for about 86.3% of total variation in brightness.
    - After accounting for the individual effects of Glass and Phosphor, combined changes in both factors account for only about 0.3% of the total variation in brightness.