Chapter 1.5 Check Your Understanding

Exercises 1–6 True or False. Give reasons.

1. The two equations $x^3 - 2x^2 - 5x = 0$ and $x^2 - 2x - 5 = 0$ have the same solution set.

Answer:

False; $x = 0$ is a solution to the first but not to the second equation.

2. The sum of all the integers in the set $\{x \mid -7 < 3x - 1 < 14\}$ is 14.

Answer:

False; $-7 < 3x - 1 < 14$, $-6 < 3x < 15$, $-2 < x < 5$, the integers in the set are -1, 0, 1, 2, 3, and 4 whose sum is 9.

3. The number -2 is in the solution set for $x^2 - 3x - 5 > |x|$.

Answer:

True; replace x by -2 and see that the resulting statement is true: $(-2)^2 - 3(-2) - 5 > |-2|$, or $5 > 2$.

4. The solution set for $(x + 3)^2 = 1$ is the same as the solution set for $x + 3 = 1$.

Answer:

False; for $(x + 3)^2 = 1$, either $x + 3 = 1$ or $x + 3 = -1$, $x = -2$ or $x = -4$. Only -2 is a solution to the second equation.

5. The solution set for $x < \frac{4}{x}$ is the same as the solution set for $x^2 < 4$.

Answer:

False; the solution set for $x < \frac{4}{x}$ is $\{x \mid x < -2 \text{ or } 0 < x < 2\}$; for $x^2 < 4$ it is $\{x \mid -2 < x < 2\}$

6. The solution set for $x < |x|$ is the set of negative numbers.

Answer:

True; since $|x| \geq 0$, if $x < 0$, then $x < |x|$. If $x \geq 0$, then $|x| = x$.

Exercises 7–10 Fill in the blank so that the resulting statement is true.

7. The largest prime number in the set $\{x \mid |x - 3| \leq 21\}$ is _____.

Answer:

$|x - 3| \leq 21$, $-21 \leq x - 3 \leq 21$, therefore $-18 \leq x \leq 24$. The largest prime number between -18 and 24 is 23.

8. The smallest positive integer that is not in the set $\{x \mid x^2 - 4x - 5 < 0\}$ is _____.

Answer:

The inequality $x^2 - 4x - 5 < 0$ is equivalent to $(x - 5)(x + 1) < 0$. Use cut points -1 and 5 to get the solution set, $\{x \mid -1 < x < 5\}$. The smallest positive integer that is not in this set is 6.

9. If $S = \{x \mid (x - 3)(x + 2) \leq 0\}$, then the sum of all integers in S is _____.

Answer:

$(x - 3)(x + 2) \leq 0$. If $x = 3$ or $x = -2$, then it is equal to zero. If you try values larger than 3 or smaller than -2, then you get numbers larger than 0. Therefore, $-2 \leq x \leq 3$ and the sum of the integers in S is $(-2) + (-1) + 0 + 1 + 2 + 3 = 3$.

10. If k is any positive number, then the number of real roots for $x^2 + 2x - k = 0$ is _____.

Answer:

Using the quadratic formula to solve $x^2 + 2x - k = 0$ gives $x = \frac{-2 \pm \sqrt{4+4k}}{2} = -1 \pm \sqrt{1+k}$. Since $k > 0$, then $-1 \pm \sqrt{1+k}$ are real numbers and so there are two roots.