Chapter 2.6 Check Your Understanding

Exercises 1–5 True or False. Give reasons.

1. If $f(x) = x^2$ and $g(x) = x^2 - 1$, then $g \circ f$ is a quadratic function in x.

Answer:

False; $(g \circ f)(x) = g(f(x)) = g(x^2) = (x^2)^2 - 1 = x^4 - 1$. Thus $(g \circ f)(x) = x^4 - 1$.

2. If $f(x) = x^2$ and g is any function for which the domain of $g \circ f$ is not the empty set, then the function $g \circ f$ must be an even function.

Answer:

True; $(g \circ f)(x) = g(x^2)$ and so $(g \circ f)(-x) = g((-x)^2) = g(x^2) = (g \circ f)(x)$. Since $(g \circ f)(-x) = (g \circ f)(x)$, then $g \circ f$ is an even function.

3. If $f(x) = 2x - 1$, then $f(a + b) = f(a) + f(b)$ for all real numbers a and b.

Answer:

False; $f(x) = 2x - 1$, then $f(a + b) = 2(a + b) - 1 = 2a + 2b - 1$, while $f(a) + f(b) = (2a - 1) + (2b - 1) = 2a + 2b - 2$.

4. If $g(x) = 3x$, then $g(c + d) = g(c) + g(d)$ for all real numbers c and d.

Answer:

True; $g(x) = 3x$, then $g(c + d) = 3(c + d) = 3c + 3d$, and $g(c) + g(d) = 3c + 3d$.

5. If $f(x) = x^2 + 1$ and $g(x) = x + 3$, then the graph of $y = (f \circ g)(x)$ contains no points below the x-axis.

Answer:

True; $f(x) = x^2 + 1$, $g(x) = x + 3$, so $(f \circ g)(x) = f(g(x)) = f(x + 3) = (x + 3)^2 + 1$. Since $(x + 3)^2 + 1 > 0$ for every x, the graph of $f \circ g$ must be above x-axis.

Exercises 6–10 Fill in the blank so that the resulting statement is true.

6. If $f(x) = 2x + 5$ and $g(x) = Int(x)$, then $(g \circ f)(\sqrt{5}) =$ ______.

Answer:

For $f(x) = 2x + 5$, $g(x) = \text{Int}(x)$, then $(g\text{o}f)\left(\sqrt{5}\right) = g\left(f\left(\sqrt{5}\right)\right) = g(x) = 4 + 5 = 9$.

7. If $f(x) = 2x + 5$ and $g(x) = Int(x)$, then $(f \circ g)(\sqrt{5}) =$ ______.

Answer:

$$(f o g)\left(\sqrt{5}\right) = f\left(g\left(\sqrt{5}\right)\right) = f\left(Int\left(\sqrt{5}\right)\right) = f(2) = 2(2) + 5 = 9$$

8. If $f(x) = 2x + 1$ and $g(x) = x^2$, then $(f \circ g)(-1) =$ ______.

Answer:

For $f(x) = 2x + 1$, $g(x) = x^2$, then $(f o g)(-1) = f(g(-1) = f(1) = 2 - 1 = 3$.

9. If $f(x) = 2x - 1$ and $g(x) = x^2 - 3x - 4$, then the zeros of $g \circ f$ are ______.

Answer:

$(g o f)(x) = g(f(x)) = g(2x - 1) = (2x - 1)^2 - 3(2x - 1) - 4 = 4x^2 - 10x = x(4x - 10)$ so the zeros are 0 and $5/2 = 2.5$.

10. If $f(x) = x^2 - 5x + 4$ and $g(x) = x^2$, then the sum of the roots of the equation $(f \circ g)(x) = 0$ is equal to ______.

Answer:

For $f(x) = x^2 - 5x + 4 = (x - 4)(x - 1)$ and $g(x) = x^2$, $(f o g)(x) = f(g(x)) = f(x^2) = (x^2 - 4)(x^2 - 1)$. The roots of $(x^2 - 4)(x^2 - 1) = 0$ are ± 2, ± 1, and the sum of the roots is 0.