Chapter 3.1 Check Your Understanding

Exercises 1–10 True or False. Give reasons. Draw a graph whenever you think it might be helpful.

1. If k is any positive number, then the graph of $y = 1 - kx^3$ contains no points in Quadrant III.

Answer:

True; for negative x, $1 - kx^3$ is positive, so the graph of f cannot have points in QIII.

2. The graph of $f(x) = x^3 + x^2 - 2x + 3$ has two turning points.

Answer:

True; draw a graph of $y = x^3 + x^2 - 2x + 3$ and see that it has two turning points.

3. If c is a zero of f, then $(0, c)$ is a point on the graph of f.

Answer:

False; a zero tells where the graph crosses the x-axis. Therefore (c, 0), not (0, c) is a point on the graph of f.

4. Every real zero of $f(x) = (1 + x^2)(x^2 - x - 2)$ is also a zero of $g(x) = x^2 - x - 2$.

Answer:

True; since $1 + x^2 > 0$ for every x, then $(1 + x)^2(x^2 - x - 2) = 0$ is equivalent to $x^2 - x - 2 = 0$.

5. The graph of $f(x) = x^3 - 3x^2 - 7x - 5$ contains points in all four quadrants.

Answer:

False; it is clear from the graph of f that it doesn't contain points in QII.

6. The degree of $f(x) = x^3 + x(1 - x^2)$ is 3.

Answer:

False; $f(x) = x^3 + x(1 - x^2) = x^3 + x - x^3 = x$ therefore the degree is one.

7. The function $f(x) = x^3 - 3x^2 - 7x + 3$ has one negative zero and two positive zeros.

Answer:

True; it is clear by looking at the graph that there is one negative zero and two positive zeros.

8. There is no fourth degree polynomial function whose graph has exactly two turning points.

Answer:

True; a fourth degree polynomial has exactly 1 or 3 turning points.

9. Using the window $[-5, 5] \times [-40, 40]$ we can conclude that $f(x) = x^3 - x^2 + 5x + 4$ has a positive zero.

Answer:

False; the window $[-5, 5] \times [-40, 40]$ shows only a negative zero, not a positive zero.

10. For $f(x) = x^3 - 18x^2 + 24x + 125$, using the window $[-8, 24] \times [-1300, 400]$ we can conclude that all zeros of f are between -3 and 20.

Answer:

True; use the given window and draw a graph of $y = x^3 - 18x^2 + 24x + 125$.