Chapter 3.2 Check Your Understanding

Draw a graph whenever helpful.

Exercises 1–6 True or False. Give reasons.

1. The function $p(x) = 4x^3 - x$ has three real zeros.

Answer:

True; $p(x) = 4x^3 - x = x(4x^2 - 1) = x(2x + 1)$. The zeros are $0, \pm\frac{1}{2}$.

2. The positive zero of $f(x) = x^3 - 3x$ is less than 1.73.

Answer:

False; $x^3 - 3x = 0$, $x(x^2 - 3) = 0$. The zeros are $0, \pm\sqrt{3}$. The positive zero $\sqrt{3}$ (≈ 1.732) is greater than 1.73.

3. For $f(x) = x^3 - 1.6x^2 - 8.52x + 15.84$, since $f(2)$ and $f(3)$ are positive, then f contains no zeros between 2 and 3.

Answer:

False; there could be a smaller interval where there is a sign change. Try $x = 2.2$.

4. The equation $2x^3 - 5x^2 + 4x - 1 = 0$ has no rational roots.

Answer:

False; $f(1) = 2 - 5 + 4 - 1 = 0$, so 1 is a rational zero.

5. The function $f(x) = (3x - 2)(x^2 - 2x - 4)$ has exactly one real zero.

Answer:

False; $f(x) = (3x - 2)(x^2 - 2x - 4)$ so $x = \frac{2}{3}$ or $x = \frac{2 \pm \sqrt{4 - 4(-4)}}{2} = 1 \pm \sqrt{5}$.

6. When $x^3 - 2x^2 + 3x - 16$ is divided by $x - 3$, then the remainder is 2.

Answer:

True; carry out the division or evaluate $x^3 - 2x^2 + 3x - 16$ at $x = 3$. In either case $r = 2$.

Exercises 7–10 Fill in the blank so that the resulting statement is true.

7. If $x^3 + 2x^2 + 1 = (x + 1)(x^2 + x - 1) + r$ for every value of x, then $r =$ _____.

Answer:

Replace x by -1 in the given equation and see that $r = 2$.

8. The number of rational zeros of $f(x) = (x^2 - 2)(x^2 - 2x + 3)$ is _____.

Answer:

$x^2 - 2 = 0$ gives $x = \pm\sqrt{2}$ (not rational), $x^2 - 2x + 3 = 0$ gives $x = 1 \pm \sqrt{2}i$. There are no rational roots.

9. The number of real roots of $(x^2 - 2)(x^2 - 2x + 3) = 0$ is _____.

Answer:

$f(x) = (x^2 - 2)(x^2 - 2x + 3)$ so $x = \pm\frac{\sqrt{4(-2)}}{2} = \pm\sqrt{2}$ or $x = 2 \pm \frac{\sqrt{4-12}}{2} = 1 \pm \sqrt{2}i$. Therefore f has two real roots, $\pm\sqrt{2}$.

10. If $x^{37} - 2x^{24} + 3x^2 - 5$ is divided by $x + 1$, then the remainder is _____.

Answer:

Substitute -1 for x in $x^{37} - 2x^{24} + 3x^2 - 5$, $(-1)^{37} - 2(-1)^{24} + 3(-1)^2 - 5 = -5$. The remainder is -5.