Chapter 3.3 Check Your Understanding

Exercises 1–7 True or False. Draw graphs when helpful.

1. The function $f(x) = x^3 - 8x^2 + 5x + 4$ has three real zeros.

Answer:

True; it is clear from the graph that there are three real zeros which are approximately -0.5, 1.2, and 7.2.

2. The graph of $f(x) = x^4 + 2x^2 + 1$ crosses the x-axis.

Answer:

False; for $f(x) = x^4 + 2x^2 + 1$, $f(x) \geq 1$ for every x.

3. The equation $x^3 - 7x + 5 = 0$ has two negative and one positive root.

Answer:

False; you can see from the graph that there is one negative and two positive roots.

4. Since $\sqrt{3}$ is a zero of $f(x) = x^3 - 2x - \sqrt{3}$, then $-\sqrt{3}$ must also be a zero.

Answer:

False; for $f(x) = x^3 - 2x - \sqrt{3}$, $f\left(\sqrt{3}\right) = 0$ but $f\left(-\sqrt{3}\right) = -2\sqrt{3}$. The conjugate-zeros theorem does not apply since all coefficients are not integers.

5. All zeros of $f(x) = x^3 - 8x^2 + 5x + 8$ lie between -1 and 8.

Answer:

True; from the graph you can find the zeros to be approximately 7.1, 1.6, and $-.7$.

6. If $f(x) = -x^4 + x^3 + 7x^2 - x - 6$, then $f(x) < 15$ for every x.

Answer:

True; draw a graph of $f(x) = -x^4 + x^3 + 7x^2 - x - 6$ and see that the maximum value of y is less than 15.

7. Based on what can be seen from the graph of $f(x) = x^3 - 40x^2 - 400x + 1600$ using $[-24, 50] \times [-5100, 17{,}000]$, we can conclude that f has one negative and two positive zeros.

Answer:

True; it is clear from the graph using $[-24, 50] \times [-5100, 17000]$.

Exercises 8–10 Fill in the blank so that the resulting statement is true.

8. For the family of functions $f(x) = x^3 + cx - 5$, the value of c for which -1 is a zero is ______.

Answer:

If -1 is a zero of $f(x) = x^3 + cx - 5$, then $f(-1) = 0$. $f(-1) = (-1)^3 - c - 5 = -c - 6$, so $-c - 6 = 0$, $c = -6$.

9. The number of real zeros of $f(x) = x^4 + 2x^3 - 2x^2 - 4x$ is ______.

Answer:

Four; by graphing f you can clearly see that there are four real zeros.

10. In applying Newton's method, if $f(x) = x^3 - 2x + 3$, then $f'(x) =$ ______.

Answer:

For $f(x) = x^3 - 2x + 3$, using the formula given on page 174, $f'(x) = 3x^2 - 2$.