Chapter 3.4 Check Your Understanding

Use a graph whenever you think it will be helpful. True or False. Give reasons.

1. If the graph of $y = \frac{1}{x}$ is translated two units left, then the resulting graph will be that of $y = \frac{1}{x - 2}$.

Answer:

False; if one graph of $y = \frac{1}{x}$ is translated 2 units to the left, the result is a graph of $y = \frac{1}{x+2}$.

2. If the graph of $y = \frac{1}{x + 2}$ is translated down one unit, then the resulting graph will be that of $y = \frac{-x - 1}{x + 2}$.

Answer:

True; $y = \frac{1}{x+2} - 1 = \frac{1-x-2}{x+2} = \frac{-x-1}{x+2}$.

3. If the graph of $y = \frac{1}{x + 2}$ is translated up one unit, then the resulting graph will be that of $y = \frac{x + 1}{x + 2}$.

Answer:

False; $y = \frac{1}{x+2} + 1 = \frac{1+x+2}{x+2} = \frac{x+3}{x+2}$.

4. The line $y = \frac{x}{2}$ is an asymptote to the graph of $y = \frac{x + 1}{2x + 1}$.

Answer:

False; the graph of $y = \frac{x+1}{2x+1}$ does not have a slant asymptote.

5. The horizontal line $y = -2$ is an asymptote to the graph of $y = \frac{1 - 2x^2}{5 + 2x + x^2}$.

Answer:

True; divide numerator and denominator by x^2: $y = \frac{\frac{1}{x^2} - 2}{\frac{5}{x^2} + \frac{2}{x} + 1}$ when $x \to \infty$, $y \to \frac{-2}{1}$.

6. The graph of $y = \dfrac{x - 2}{x^2 - x + 2}$ has no vertical asymptotes.

Answer:

True; the denominator of $y = \frac{x-2}{x^2-x+2}$ is not equal to zero for any value of x.

Exercises 7–8 *Suppose* $f(x) = \dfrac{3x^2 + 1}{x^2 + 1}$.

7. There is no value of x for which $f(x) = 3$.

Answer:

True; since the degrees of the numerator and denominator are both 2 (m = n), the graph has a horizontal asymptote, $y = \frac{a}{b} = \frac{3}{1} = 3$.

8. For every real number x, $f(x)$ is in the interval $[1, 3)$.

Answer:

True; if f(x) = 3, then $3 = \frac{3x^2+1}{x^2+1}$, $3x^2 + 3 = 3x^2 + 1$ and we get no solutions.

9. If $f(x) = \dfrac{x^2 + 100}{x}$, then the graph of f has a local minimum point in the third quadrant.

Answer:

False; it is clear from the graph that there is not a local minimum in QIII.

10. The graph of $f(x) = \dfrac{2x^3 - 3x^2 + 500}{x^4 + 8x + 50}$, has one zero and no vertical asymptotes.

Answer:

True; draw a graph of $y = 2x^3 - 3x^2 + 500$ and see that it crosses the x-axis at only on point. Draw a graph of $y = x^4 + 8x + 50$ and see that it does not cross the x-axis at any point.