Chapter 4.2 Check Your Understanding

Exercises 1–5 True or False. Give reasons.

1. $\log_3 7 < \log_2 7$ (*Hint:* Think exponents.)

Answer:

True; $1 < \log_3 7 < 2$, $2 < \log_2 7 < 3$, hence $\log_3 7 < \log_2 7$.

2. $\log_5 \pi < \log_2 \pi$

Answer:

True; $0 < \log_5 \pi < 1$, $1 < \log_2 \pi < 2$, hence $\log_5 \pi < \log_2 \pi$.

3. If $f(x) = \log_2 x$, then $f^{-1}(x) = 2^x$.

Answer:

True; for $y = \log_2 x$, interchange x and y and solve for y: $x = \log_2 y$, then $2^x = y$, or $f^{-1}(x) = 2^x$.

4. The domain of $g(x) = \log_2 x^2$ is the set of all real numbers.

Answer:

False; the number 0 is not in the domain.

5. The domain of $g(x) = \log_2(x^2 + 1)$ is the set of all real numbers.

Answer:

True; $x^2 + 1 > 0$ for every x.

Exercises 6–10 Fill in the blank so that the resulting statement will be true.

6. The graphs of $y = 2^{-x}$ and $x + y = 3$ intersect in Quadrant(s) _____.

Answer:

Draw graphs of $y_1 = 2^{-x}$ and $y_2 = 3 - x$ and see that they intersect in QI and QII.

7. The graphs of $y = 3^{-x}$ and $x - y + 3 = 0$ intersect in Quadrant(s) _____.

Answer:

QII; it is clear from the graphs that they intersect approximately at the point $(-0.74, 2.3)$ in QII.

8. The number of points of intersection of the graph of $y = \log_2 x$ and $y = 4 - x^2$ is _____.

Answer:

Draw graphs of $y_1 = \log_2 x$ and $y_2 = 4 - x^2$ and see that there is only one point of intersection.

9. A point on both of the graphs of $y = 2^x$ and $y = 3^x$ is _____.

Answer:

(0, 1); you can find this by the intersection of the graphs or simply by observing that $2^0 = 1 = 3^0$.

10. The graphs of $y = -\ln x$ and $y = e^{x-2}$ intersect in Quadrant _____.

Answer:

Draw graphs of $y_1 = -\ln x$ and $y_2 = e^{x-2}$ and see that they intersect in QI.