Chapter 4.3 Check Your Understanding

Exercises 1–5 True or False. Give reasons.

1. $\log_5(\sqrt{2} + \sqrt{3}) = \frac{1}{2}(\log_5 2 + \log_5 3)$.

Answer:

False; $\log_5\left(\sqrt{2}+\sqrt{3}\right) \neq \log_5\sqrt{2}+\log_5\sqrt{3}$.

2. The graph of $y = -2^x$ and $y = -\log_2(x + 2)$ intersect at $(0, -1)$.

Answer:

True; for $y = -2^x$, when $x = 0$, $y = -2^0 = -1$; for $y = -\log_2(x+2)$, when $x = 0$, $y = -\log_2(2) = -1$. Therefore $(0, -1)$ is on both graphs.

3. The graph of $y = \log_4(4x)$ is the same as the graph of $y = 1 + \log_4 x$.

Answer:

True; $\log_4(4x) = \log_4 4 + \log_4 x = 1 + \log_4 x$.

4. The graph of $y = \log_2(4x)$ can be drawn by translating the graph of $y = \log_2 x$ up 2 units.

Answer:

True; $\log_2(4x) = \log_2 4 + \log_2 x = 2 + \log_2 x$.

5. For every real number x, $\log_2(2x) = 1 + \log_2 x$.

Answer:

False; $\log_2(2x)$ and $\log_2 x$ are undefined when x is negative.

Exercises 6–10 Fill in the blank so that the resulting statement will be true.

6. The domain of $f(x) = \log_2(x + 2) + \log_2(1 - x)$ is _____.

Answer:

For $f(x) = \log_2(x+2) + \log_2(1-x)$, $D = \{x | x+2 > 0 \text{ and } 1-x > 0\} = \{x | x > -2 \text{ and } x < 1\}$ $= \{x | -2 < x < 1\}$.

7. The sum of all the prime numbers between $\log_2 0.5$ and $\log_2 256$ is _____.

Answer:

17; $\log_2 0.5 = \log_2 2^{-1} = -1$ and $\log_2 256 = \log_2 2^8 = 8$ and therefore $2+3+5+7 = 17$.

8. If $f(x) = \log_4 x$ then $f^{-1}(x) =$ _____.

Answer:

If $f(x) = \log_4 x$ then we can get a formula for f^{-1} by interchanging x and y in $y = \log_4 x$ and solving for y: $x = \log_4 y$, $y = 4^x$, so $f^{-1}(x) = 4^x$.

9. The graphs of $y = -3^x$ and $y = \log_3 x$ intersect in Quadrant _____.

Answer:

Four; it is clear from the graphs that they intersect in QIV.

10. The integers between $\log_2 1$ and $\log_2 128$ are _____.

Answer:

$\log_2 1 = 0$ and $\log_2 128 = \log_2(2^7) = 7$. The integers between 0 and 7 are 1,2,3,4,5,6.