Chapter 4.4 Check Your Understanding

Exercises 1–5 True or False. Give reasons.

1. $\log 16 < \ln 5$

Answer:

True; $\log 16 \approx 1.20$, $\ln 5 \approx 1.61$, so $\log 16 < \ln 5$.

2. $\ln(\sqrt{2} + \sqrt{5}) = \frac{1}{2}(\ln 2 + \ln 5)$

Answer:

False; $\ln(\sqrt{2} + \sqrt{5} \approx 1.29$, $\frac{1}{2}(\ln 2 + \ln 5) \approx 1.15$.

3. For all positive numbers c and d,
$\ln(c + d) = \ln c + \ln d$.

Answer:

False; try $c = 1$ and $d = 1$. Then $\ln (c + d) = \ln (1 + 1) = \ln 2 \approx 0.69$, but $\ln c + \ln d = \ln 1 + \ln 1 = 0 + 0 = 0$

4. The graph of $y = \log x$ is above the graph of $y = \ln x$ for all $x > 1$.

Answer:

False; if $x > 1$, then $\log x < \ln x$.

5. The graph of $y = \ln x$ is above the graph of $y = \log_3 x$ for every $x > 1$.

Answer:

True; it is clear by graphing $y = \ln x$ and $y = \log_3 x$ on the same screen.

Exercises 6–10 Fill in the blank so that the resulting statement is true.

6. The number of integers between $\ln 4$ and $5\ln 25$ is _____.

Answer:

$\ln 4 \approx 1.39$, $5\ln 25 \approx 16.09$, so the integers between $\ln 4$ and $5\ln 25$ are 2, 3, 4, ... 16; fifteen of them.

7. The sum of the integers between $\ln 4$ and $2\ln 25$ is _____.

Answer:

The answer is 20; $\ln 4 \approx 1.39$, $2\ln 25 \approx 6.4$, so $2 + 3 + 4 + 5 + 6 = 20$.

8. If $S = \{x \mid \ln 0.5 \leq x \leq 5\ln 25\}$, then the smallest positive integer that is not in S is _____.

Answer:

$S = \{x \mid \ln 0.5 \leq x \leq 5 \ln 25\} \approx \{x \mid -0.69 \leq x \leq 16.09\}$ and so the smallest positive integer that is not in S is 17.

9. The graph of $y = \ln(x^3 + x^2 - 4x + 6)$ has turning points in Quadrants _____.

Answer:

QI and QII; it is clear from the graph of $y = \ln(x^3 + x^2 - 4x + 6)$ that turning points occur at about $(-1.54, 2.39)$ and $(0.87, 1.37)$.

10. The local minimum point (2 decimal places) for the graph of $y = 2 + \ln(x^3 + x^2 - 4x + 4)$ is _____.

Answer:

Draw a graph of $y = 2 + \ln(x^3 + x^2 - 4x + 4)$ and zoom in on the local minimum point (0.87, 2.66).