Chapter 8.1 Check Your Understanding

Exercises 1–5 True or False. Give reasons.

1. $\sum_{k=1}^{147} (-1)^k = 1.$

Answer:

False; $\sum_{k=1}^{147}(-1)^k = -1+1-1+\ldots+1-1 = -1.$

2. $\sum_{k=1}^{4} k^2 = \left(\sum_{k=1}^{4} k\right)^2.$

Answer:

False; $\sum_{k=1}^{4} k^2 = 1^2+2^2+3^2+4^2 = 30$, $(\sum_{k=1}^{4} k)^2 = (1+2+3+4)^2 = 100.$

3. The sequence given by $b_n = 2n + 9 - n^2$ contains only positive integers.

Answer:

False; for $b_n = 2n+9-n^2$, $b_5 = 10+9-25 = -6.$

4. For every positive integer n, $(n+4)! = n! + 4!$.

Answer:

False; a counterexample is $n = 1$: $(1+4)! = 5! = 120$, where as $1! + 4! = 1 + 24 = 25.$

5. $\sum_{k=1}^{4} \frac{1}{k} = \sum_{j=2}^{5} \frac{1}{j-1}.$

Answer:

True; both summations are $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$.

Exercises 6–10 Fill in the blank so that the resulting statement is true.

6. If $a_n = 2n + 1$, then $a_5 - a_2 =$ _____ .

Answer:

$a_n = 2n + 1$, then $a_5 - a_2 = (2 \cdot 5 + 1) - (2 \cdot 2 + 1) = 11 - 5 = 6$.

7. If $b_n = \dfrac{n! + 4}{n!}$, then $b_4 =$ _____.

Answer:

$b_n = \frac{n!+4}{n!}$, then $b_4 = \frac{4!+4}{4!} = \frac{24+4}{24} = \frac{28}{24} = \frac{7}{6}$.

8. If $a_n = 8 - 2n$, then $a_5 =$ _____.

Answer:

$a_n = 8 - 2n$, then $a_5 = 8 - 2 \cdot 5 = 8 - 10 = -2$.

9. $\sum_{k=1}^{4} (k - 2)^2 =$ _____.

Answer:

$\sum_{k=1}^{4}(k-2)^2 = (1-2)^2 + (2-2)^2 + (3-2)^2 + (4-2)^2 = 1 + 0 + 1 + 4 = 6$.

10. If $a_n = (-1)^n(2n - 1)$, then $\sum_{k=1}^{4} a_k =$ _____.

Answer:

$a_n = (-1)^n(2n-1)$, then $\sum_{k=1}^{4} a_k = \sum_{k=1}^{4}(-1)^k(2k-1) = -(2 \cdot 1 - 1) + (2 \cdot 2 - 1) - (2 \cdot 3 - 1) + (2 \cdot 4 - 1) = -1 + 3 - 5 + 7 = 4$.