

Chapter 8.2 Check Your Understanding

It will be helpful to use the Technology Tip (page 444) to get the first several terms of $\{a_n\}$

Exercises 1–5 True or False. Give reasons. Use sequence $\{a_n\}$ defined by $a_1 = 1$ and

$$a_{n+1} = 1 + \frac{1}{a_n}$$

1. Every term of $\{a_n\}$ is less than or equal to 2.

Answer:

Exercises 1-5 The sequence $\{a_n\}$ is defined by $a_1 = 1$ and $a_{n+1} = 1 + \frac{1}{a_n}$ for $n \geq 1$. The first few terms are 1, $2, 3/2, 5/3$ (≈ 1.667), $8/5$ ($= 1.6$), $13/8$ ($= 1.625$), $21/13$ (≈ 1.615), $34/21$ (≈ 1.619), $55/34$ (≈ 1.6176).

True; this is true for the terms listed above. In general, if $1 \leq a_n \leq 2$, then $a_{n+1} = 1 + \frac{1}{a_n} \leq 2$.

2. The sequence is decreasing; that is $a_{n+1} < a_n$ for every n .

Answer:

False; see terms listed above.

3. The even-numbered terms are greater than the odd-numbered terms.

Answer:

True; based on the terms listed above, the given statement appears to be true.

4. The subsequence consisting of the odd-numbered terms, $\{a_1, a_3, a_5, \dots\}$, is decreasing.

Answer:

False; see the terms listed above.

5. The subsequence consisting of the even-numbered terms, $\{a_2, a_4, a_6, \dots\}$, is increasing.

Answer:

False; the even numbered terms begin $2, \frac{5}{3}, \frac{13}{8}, \frac{34}{21}$ and so clearly $\{a_{2n}\}$ is not increasing.

Exercises 6–10 Fill in the blank so that the resulting statement is true.

Exercises 6–8 Sequence $\{a_n\}$ is defined by $a_1 = 1$ and $a_{n+1} = a_n + 4$ for $n > 1$.

6. The smallest integer greater than a_5 is _____.

Answer:

Exercises 6-8 Given $a_1 = 1$ and $a_{n+1} = a_n + 4$. The first few terms are 1, 5, 9, 13, 17, 21, 28, ...

$a_5 = 17$ and the integers greater than 17 are 18, 19, 20, ...; the smallest of these is 18.

7. The number of terms of $\{a_n\}$ between 8 and 20 is _____.

Answer:

Three; $a_3 = 9$, $a_4 = 13$, $a_5 = 17$, and so there are three terms between 8 and 20.

8. The sum of the first 5 terms is _____.

Answer:

$$a_1 + a_2 + a_3 + a_4 + a_5 = 1 + 5 + 9 + 13 + 17 = 45.$$

Exercises 9–10 Sequence $\{b_n\}$ is defined by $b_1 = 1$ and $b_{n+1} = \sqrt{1 + b_n^2}$ for $n \geq 1$.

9. $b_5 =$ _____.

Answer:

Exercises 9-10 Sequence $\{b_n\}$ is defined by $b_1 = 1$, $b_{n+1} = \sqrt{1 + b_n^2}$ for $n \geq 1$. The first few terms are 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, ...

$$b_5 = \sqrt{5}$$

10. The smallest prime number that is greater than b_5 is _____.

Answer:

$b_5 = \sqrt{5} \approx 2.236$ and so the smallest prime greater than b_5 is 3.