Chapter 8.3 Check Your Understanding

Exercises 1–6 True or False. Give reasons.

1. If $\{a_n\}$ is an arithmetic sequence, then $a_6 - a_3 = a_8 - a_5$.

Answer:

True; $a_6 - a_3 = (a_1 + 5d) - (a_1 + 2d) = 3d$, $a_8 - a_5 = (a_1 + 7d) - (a_1 + 4d) = 3d$.

2. The sequence beginning $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$ could be an arithmetic sequence.

Answer:

False; $\frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$ while $\frac{1}{6} - \frac{1}{4} = -\frac{1}{12}$.

3. If $\{c_n\}$ is a geometric sequence, then $\frac{c_5}{c_2} = r^3$.

Answer:

True; $c_5/c_2 = ar^4/ar = r^3$.

4. The sequences $\{a_n\}$ and $\{b_n\}$ given by $a_n = 2n$ and $b_n = \log(100^n)$ are identical.

Answer:

True; $a_n = 2n$ and $b_n = \log 100^n = n \log 100 = n \cdot 2 = 2n$.

5. In a geometric sequence if the common ratio is negative, then after a certain point in the sequence, all the terms will be negative.

Answer:

False; if r is negative then the terms alternate in sign.

6. In an arithmetic sequence if the common difference is negative, then after a certain point in the sequence, all the terms will be negative.

Answer:

True; $a_n = a_1 + nd$, and so if $d < 0$, there is an n such that from some point on a_n must be negative.

Exercises 7–10 Fill in the blank so that the resulting statement is true.

7. $14 + \sum_{k=1}^{5} (-2)^k = \_\_\_\_\_$.

Answer:

$$14 + \sum_{k=1}^{5} (-2)^k = 14 + (-2 + 4 - 8 + 16 - 32) = -8.$$

8. $\sum_{k=1}^{15} (8 - k) = \_\_\_\_\_$.

Answer:

$$\sum_{k=1}^{15} (8 - k) = 7 + 6 + 5 + \ldots + (-7) = 0.$$

9. $0.999\ldots = 0.\overline{9} = \_\_\_\_\_$.

Answer:

$$0.999\ldots = 0.9 + 0.09 + 0.009 + \ldots = \frac{0.9}{1-0.1} = \frac{0.9}{0.9} = 1$$

10. $11(0.727272\ldots) = 11(0.\overline{72}) = \_\_\_\_\_$.

Answer:

$$11(0.\overline{72}) = 11(0.72 + 0.0072 + 0.000072 + \ldots) = 11\left(\frac{.72}{1-0.01}\right) = 11\left(\frac{.72}{.99}\right) = 11\left(\frac{72}{99}\right) = \frac{72}{9} = 8.$$