Chapter 8.4 Check Your Understanding

Exercises 1–10 True or False. Give reasons.

1. When n is 1, 2, 3, 4, or 5, the sum of the first n odd positive integers is equal to n^2.

Answer:

True; $1 = 1^2$, $1 + 3 = 2^2$, $1 + 3 + 5 = 3^2$, $1 + 3 + 5 + 7 = 4^2$, $1 + 3 + 5 + 7 + 9 = 5^2$.

2. When n is 1, 2, 3, 4, or 5, the sum of the first n even positive integers is equal to $n(n + 1)$.

Answer:

True; $2 = 1(1 + 1)$, $2 + 4 = 2(2 + 1)$, $2 + 4 + 6 = 3(3 + 1)$, $2 + 4 + 6 + 8 = 4(4 + 1)$, $2 + 4 + 6 + 8 + 10 = 5(5 + 1)$.

3. If $f(n) = n^2 - n + 17$, then $f(n)$ is a prime number for $n = 1, 2, 4, 8$, and 17.

Answer:

False; for $n = 17$, $n^2 - n + 17 = 17^2 - 17 + 17 = 17^2$.

4. If $f(n) = n^2 + n$, then $f(n)$ is an even number for every positive integer n.

Answer:

True; $f(n) = n^2 + n = n(n + 1)$. Since n and n + 1 are consecutive integers, then one must be even and the other odd, and so the product must be even.

5. Evaluating the expressions $(n + 1)^2$ and 2^n for $n = 1, 2, 3, 4, 5$, and 6, it is reasonable to conclude that $(n + 1)^2 > 2^n$ for every positive integer.

Answer:

False; for $n = 6$, $(6 + 1)^2 > 2^6$, or $49 > 64$ is false.

6. For every positive integer n, $3^n + 1$ is an even number.

Answer:

True; 3^n is odd for every n and so $3^n + 1$ must be even.

7.	For every positive integer n, the units digit of $5^n - 1$ is 4. **Answer:** True; the units digit for 5^n is 5 for every n, so the units digit for $5^n - 1$ must be 4.
8.	When n is 1, 2, 3, or 4, $5^n + 1$ is not divisible by 4. **Answer:** True; evaluate $5^n + 1$ for $n = 1, 2, 3$, and 4. None of the resulting numbers is divisible by 4.
9.	For every positive integer n, the units digit of 2^n is 2, 4, or 8. **Answer:** False; when n is 4, $2^n = 2^4 = 16$.
10.	For every positive integer n, the units digit of $4^n - 1$ is 3 or 5. **Answer:** True; the units digit of 4^n is either 4 or 6 for every positive integer n. Therefore, the units digit of $4^n - 1$ must be 3 or 5.