Chapter 8.5 Check Your Understanding

Exercises 1–10 True or False. Give reasons.

1. There is no positive integer n such that $(n+1)! = n! + 1!$.

Answer:

False; when n is 1, $(1+1)! = 1! + 1$, $2! = 1 + 1$, $2 = 2$.

2. There is no positive integer n such that $n^2 + n = 6$.

Answer:

False; when n is 2, $2^2 + 2 = 6$, $4 + 2 = 6$.

3. For every positive integer n, $(n+1)^2 \geq 2^n$.

Answer:

False; when $n \geq 6$, then $(n+1)^2 < 2^n$.

4. For every positive integer n, $\sin n\pi = 0$.

Answer:

True; from the definition of the sine function in section 5.2, $P(n\pi) = (\pm 1, 0)$ and so $\sin n\pi = 0$.

5. For every positive integer n, $(2n-1)(2n+1)$ is an odd number.

Answer:

True; $2n - 1$ and $2n + 1$ are odd numbers for every n, and so their product is odd.

6. For every positive integer n, $n^2 + n$ is an even number.

Answer:

True; $n^2 + n = n(n+1)$ and since n and $n + 1$ are consecutive integers, on of them must be even.

7. For every positive integer n, $n^2 + 1 \geq 2n$.

Answer:

True; $(n-1)^2 \geq 0$ and so $n^2 - 2n + 1 \geq 0$, or $n^2 + 1 \geq 2n$.

8. For every positive integer n, $(n + 1)^3 - n^3 - 1$ is divisible by 6.

Answer:

True; $(n+1)^3 - n^3 - 1 = (n^3 + 3n + 3n + 1) - n^3 - 1 = 3n^2 + 3n = 3n(n+1)$. Since $n(n+1)$ is even, $3n(n+1)$ must be divisible by 6.

9. For every positive integer n, $n^2 - n + 17$ is a prime number.

Answer:

False; when n is 17, $n^2 - n + 17 = 17^2 - 17 + 17^2 = 17^2$.

10. For every integer n greater than 1, $\log_2 n \geq \log_n 2$.

Answer:

True; $\log_2 n = \frac{\ln n}{\ln 2}$ and $\log_n 2 = \frac{\ln 2}{\ln n}$. Since $\frac{\ln n}{\ln 2} \geq \frac{\ln 2}{\ln n}$ for every integer n greater than 1, then $\log_2 n \geq \log_n 2$.