

Chapter 8.6 Check Your Understanding

Exercises 1–6 True or False. Give reasons.

1. For every positive integer n , $(3n)! = (3!)(n!)$.

Answer:

False; true for $n = 1$ but not when $n = 2$; $(3 \cdot 2)! = (3!)(2!)$, $6! = (6)(2)$, $120 = 12$ is a false statement.

2. There are ten terms in the expression of $(1 + x)^{10}$.

Answer:

False; when $(1 + x)^{10}$ is expanded there are eleven terms.

3. The middle term of the expansion of $(x + \frac{1}{x})^8$ is 70.

Answer:

True; the expansion of $(x + \frac{1}{x})^8$ has 9 terms, so the middle term is given by $\binom{8}{4}(x^4)(\frac{1}{x})^4$
 $= \binom{8}{4} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4} = 70$.

4. The expansion of $(x^2 + 2x + 1)^8$ is the same as the expansion of $(x + 1)^{16}$.

Answer:

True; $(x^2 + 2x + 1)^8 = [(x + 1)^2]^8 = (x + 1)^{16}$.

5. $\binom{8}{1} + \binom{8}{2} - \binom{9}{2} = 0$.

Answer:

True; $\binom{8}{1} + \binom{8}{2} - \binom{9}{2} = 8 + \frac{8 \cdot 7}{1 \cdot 2} - \frac{9 \cdot 8}{1 \cdot 2} = 8 + 28 - 36 = 0$

6. For every positive integer x , $(\sqrt{x} + \frac{1}{x})^4 = x^2 + \frac{1}{x^4}$.

Answer:

False; the expansion of $(\sqrt{x} + \frac{1}{x})^4$ has 5 terms and so it cannot equal $x^2 + \frac{1}{x^4}$.

Exercises 7–10 Fill in the blank so that the resulting statement is true.

7. After simplifying the expansion of $(x^2 - \frac{1}{x})^5$, the coefficient of x^4 is _____.

Answer:

$(x^2 - \frac{1}{x})^5 = \sum_{k=0}^5 \binom{5}{k} (x^2)^{5-k} (-\frac{1}{x})^k = \sum_{k=0}^5 (-1)^k \binom{5}{k} x^{10-3k}$. For the term involving x^4 , $10 - 3k = 4$, $3k = 6$, $k = 2$. The coefficient of x^4 is $(-1)^2 \binom{5}{2} = 10$.

8. In the expansion of $(\sqrt{x} - \frac{1}{\sqrt{x}})^6$, the middle term is _____.

Answer:

For $(\sqrt{x} - \frac{1}{\sqrt{x}})^6$, the middle term the four on, or $\binom{6}{3} (\sqrt{x})^{6-3} (\frac{-1}{\sqrt{x}})^3 = -\binom{6}{3} = -20$.

9. $\binom{8}{3} - \binom{8}{2} =$ _____.

Answer:

$$\binom{8}{3} - \binom{8}{2} = \frac{8 \cdot 7 \cdot 6}{3!} - \frac{8 \cdot 7}{2!} = 56 - 28 = 28.$$

10. The number of terms in the expansion of $(x^2 + 4x + 4)^{12}$ is _____.

Answer:

$(x^2 + 4x + 4)^{12} = ((x + 2)^2)^{12} = (x + 2)^{24}$. Thus there are 25 terms.