Chapter 9.2 Check Your Understanding

Exercises 1–6 True or False. Give reasons.

1. The matrix for the system

$$\begin{aligned} 2x - y &= 5 \\ x + 2y &= 3 \end{aligned}$$

is

$$\begin{bmatrix} 2 & -1 & 5 \\ 1 & 2 & 3 \end{bmatrix}.$$

Answer:

True; the coefficients and constant terms appear in the correct order.

2. The system of linear equations that correspond to the matrix

$$\begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

is

$$\begin{aligned} 3x - y &= 0 \\ y &= 0. \end{aligned}$$

Answer:

True; the coefficients and constant terms match the entries in the matrix.

3. For the system of equations in x and y that correspond to the matrix

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 3 & 6 \end{bmatrix},$$

the solution is given by $x = 1$, $y = 2$.

Answer:

True; $x = 1$ and $y = 2$ satisfy both equations $2x - y = 0$ and $0 \cdot x + 3y = 6$.

4. The systems of linear equations that correspond to the following matrices are equivalent.

$$\begin{bmatrix} 1 & -2 & -3 & 3 \\ 0 & 1 & -9 & 10 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Answer:

True; the elementary transformations $2E_2 + E_1 \to E_1$, $21E_3 + E_1 \to E_1$, and $9E_3 + E_2 \to E_2$ will transform the first matrix into the second matrix.

5. The triangle formed by the three lines $2x - 3y = 1$, $2x + 3y = 4$, and $3x + 2y = 3$ is a right triangle. (*Hint:* Consider the slopes of the lines.)

Answer:

True; let L_1, L_2, L_3 denote the three lines. The slope of L_1 is $\frac{2}{3}$ and the slope of L_3 is $-\frac{3}{2}$. Therefore, L_1 and L_3 are perpendicular to each other.

6. The triangle formed by the three lines $x + 2y = 3$, $2x - 2y = 5$, and $x - 2y = 4$ is a right triangle. (*Hint:* Consider the slopes of the lines.)

Answer:

False; let L_1, L_2, L_3 denote the three lines. The slopes of L_1, L_2, L_3 are $-\frac{1}{2}, 1, \frac{1}{2}$, and so we do not have perpendicular lines.

Exercises 7–10 Fill in the blank so that the resulting statement is true. Solve the system of equations that corresponds to the matrix.

7. $\begin{bmatrix} 1 & -1 & 3 \\ 0 & -1 & 2 \end{bmatrix}$; solution is ______.

Answer:

$\begin{bmatrix} 1 & -1 & 3 \\ 0 & -1 & 2 \end{bmatrix}$. From R_2, $-y = 2$, $y = -2$. Substituting -2 for y in $x - y = 3$ we get $x = 1$. The solution is $x = 1, y = -2$.

8. $\begin{bmatrix} 2 & 0 & -4 \\ 1 & 2 & 6 \end{bmatrix}$; solution is ______.

Answer:

$\begin{bmatrix} 2 & 0 & -4 \\ 1 & 2 & 6 \end{bmatrix}$. From R_1, $2x = -4$, $x = -2$. Substituting -2 for x in $x + 2y = 6$ we get $y = 4$. The solution is $x = -2, y = 4$.

9. $\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & -3 & 6 \end{bmatrix}$; solution is ______.

Answer:

$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & -3 & 6 \end{bmatrix}$. From R_3, $-3z = 6$, $z = -2$. From R_2, $y - z = 4$, $y = 4 + z = 4 + (-2) = 2$.
From R_1, $x - 2y = -3$, $x = 2y - 3 = 2(2) - 3 = 1$. The solution is $x = 1$, $y = 2$, $z = -2$.

10. $\begin{bmatrix} 1 & 0 & -1 & 4 \\ -1 & 0 & 0 & -3 \\ 0 & 2 & -1 & 5 \end{bmatrix}$; solution is ______.

Answer:

$\begin{bmatrix} 1 & 0 & -1 & 4 \\ -1 & 0 & 0 & -3 \\ 0 & 2 & -1 & 5 \end{bmatrix}$. From R_2, $-x = -3$, then $x = 3$. From R_1, $x - z = 4$, $z = x - 4 = 3 - 4 = -1$.

From R_3, $2y - z = 5$, $y = \frac{z+5}{2} = \frac{-1+5}{2} = 2$. The solution is $x = 3$, $y = 2$, and $z = -1$.