Chapter 9.5 Check Your Understanding

Exercises 1–6 True or False. Give reasons.

1. The determinant of

$$\begin{bmatrix} 2 & -1 \\ 3 & -5 \end{bmatrix}$$

is equal to -7.

Answer:

True; $D = -10 + 3 = -7$.

2. The only solution of the equation

$$\begin{vmatrix} x & -2 \\ 4 & 2 \end{vmatrix} = 6$$

is given by $x = -1$.

Answer:

True; $2x + 8 = 6$, $2x = -2$, $x = -1$.

3.

$$\begin{vmatrix} 1 & 3 & -2 \\ 0 & 1 & 4 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix}.$$

Answer:

True; expand by the elements of the first column.

4. The solution of the equation

$$\begin{vmatrix} x & 3 & -2 \\ 0 & 1 & 4 \\ 0 & 1 & 1 \end{vmatrix} = 3$$

is given by $x = -1$.

Answer:

True; expand by the elements of the first column and then solve for x, $-3x = 3$, $x = -1$.

5. The solution set for the equation

$$\begin{vmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{vmatrix} = 1$$

is the empty set.

Answer:

False; the equation is equivalent to $\sin^2 x + \cos^2 x = 1$, and so the solution set is $\mathbb{R}$.

6. If every element of 2×2 matrix A is a positive number, then the determinant of A is a positive number.

Answer:

False; a counterexample is $\begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix} = -7$.

Exercises 7–10 Fill in the blank so that the resulting statement is true. All questions refer to the matrix

$$A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 0 & 2 \end{bmatrix}.$$

7. The determinant of A is equal to _____.

Answer:

Evaluate by the first row. $\begin{vmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} = (2-1) + (1) = 2.$

8. The minor M_{31} is equal to _____.

Answer:

$$M_{31} = \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = 1 - 1 = 0.$$

9. The cofactor C_{11} is equal to _____.

Answer:

$$C_{11} = \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = 2$$

10. The cofactor C_{12} is equal to _____.

Answer:

$$C_{12} = (-1)\begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix} = (-1)(2-1) = -1.$$