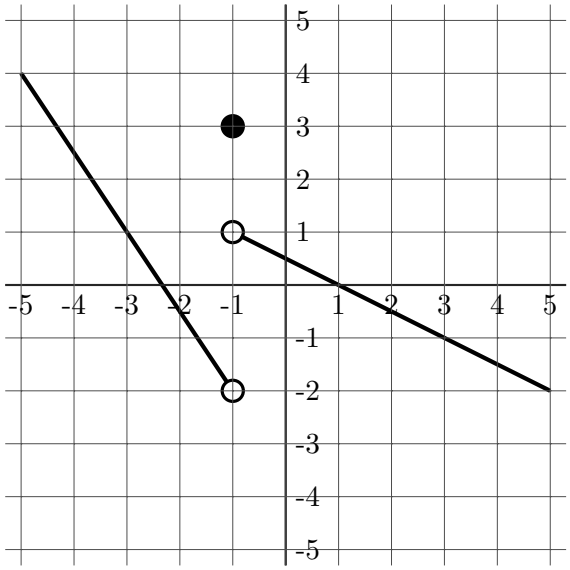
1. The graph below is the function $f(x)$. Answer the following questions. Write DNE for undefined or Does Not Exist



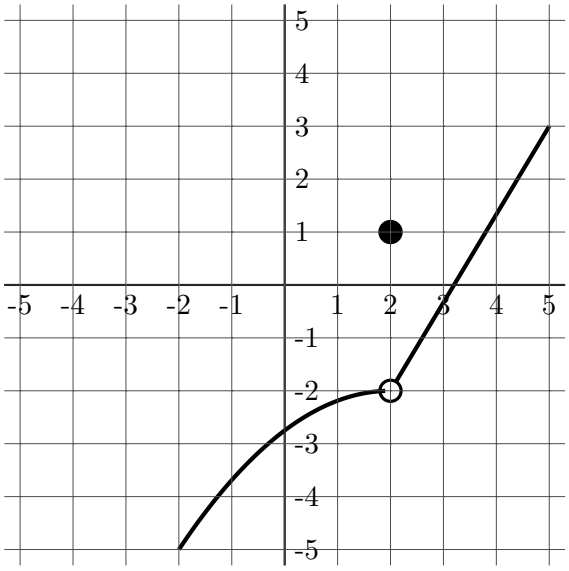

(a) Find $\lim_{x \to -1^-} f(x)$.

(b) Find $\lim_{x \to -1^+} f(x)$.

(c) Find $\lim_{x \to -1} f(x)$.

(d) Find $f(-1)$

2. The graph below is the function $f(x)$. Answer the following questions. Write DNE for undefined or Does Not Exist



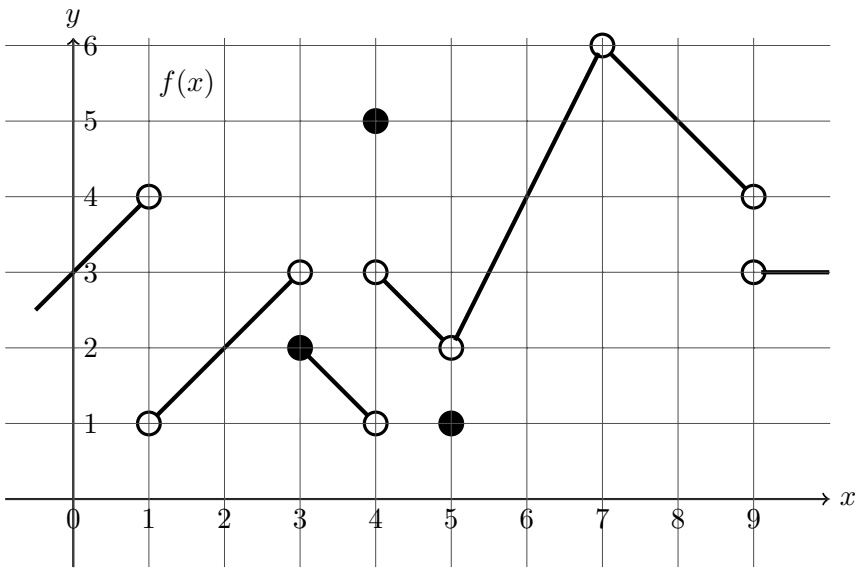

(a) Find $\lim_{x \to 2^-} f(x)$.

(b) Find $\lim_{x \to 2^+} f(x)$.

(c) Find $\lim_{x \to 2} f(x)$.

(d) Find $f(2)$

3. The graph below is the function $f(x)$. Answer the following questions. Write DNE for undefined or Does Not Exist.




(a) Find $f(3)$.

(b) Find $\lim_{x\to 5^-} f(x)$.

(c) Find $\lim_{x\to 9^+} f(x)$.

(d) Find $\lim_{x\to 3} f(x)$.

(e) Find $\lim_{x\to 4^+} \dfrac{f(x)-1}{f(x+3)}$.

(f) Find $\lim_{x\to 3^-} f(f(x)+1)$.

(g) Find $\lim_{h\to 0} \dfrac{f(3.5+h)-f(3.5)}{h}$.

4. Evaluate the limit $\lim_{x \to 3} 7x^5 - 6x$.

5. Evaluate the limit $\lim_{x \to -4} \dfrac{x^2 + 12x + 32}{x + 4}$.

6. Evaluate the limit $\lim_{x \to -3} \dfrac{-7x - 21}{x^2 + 4x + 3}$.

7. Let

$$f(x) = \begin{cases} 2 - x - x^2 & \text{if} \quad x \leq 2 \\ 2x - 3 & \text{if} \quad x > 2 \end{cases}$$

Calculate the following limits. Write "DNE" if the limit does not exist.

(a) $\lim_{x \to 2-} f(x) =$

(b) $\lim_{x \to 2+} f(x) =$

(c) $\lim_{x \to 2} f(x) =$

8. Evaluate $\lim_{x \to b} \dfrac{\frac{1}{b} - \frac{1}{6}}{b - 6}$.

9. Evaluate $\lim_{x \to 1} 6$.

10. Evaluate $\lim_{x \to -2} x$.

11. Evaluate $\lim_{y \to 5} 3y + 7$.

12. Evaluate $\lim_{h \to 0} \dfrac{(-2+h)^2 - 4}{h}$.

## Answer Key

**1** **(a)** $-2$ **(b)** 1 **(c)** A limit does not exist. **(d)** 3

**2** **(a)** $-2$ **(b)** $-2$ **(c)** $-2$ **(d)** 1

**3** **(a)** 2 **(b)** 2 **(c)** 3 **(d)** DNE **(e)** $\frac{1}{3}$ **(f)** 1 **(g)** $-1$

**4** 1683

**5** 4

**6** $\frac{7}{2}$

**7** **(a)** $-4$ **(b)** 1 **(c)** DNE

**8** $-\frac{1}{36}$

**9** 6

**10** $-2$

**11** 22

**12** $-4$

## Detailed Solutions

**1** (a) As $x$ gets **very close to −1** from the **left**, the value of the function $f(x)$ is getting closer to $-2$.

(b) As $x$ gets **very close to −1** from the **right**, the value of the function $f(x)$ is getting closer to 1.

(c) Since the limit from the **left** and limit from the **right** are different, $\lim_{x\to -1} f(x)$ does not exist.

(d) $f(-1) = 3$. Note that $f(-1) \neq -2$ and $f(-1) \neq 1$, since there are holes at $(-1,-2)$ and $(-1,1)$ respectively.

**2** (a) As $x$ gets very close to **−1** from the **left**, the values of $y = f(x)$ approach $-2$.

(b)

**3** Critical numbers occur when the derivative is zero. We take the derivative of $f(x)$ using the power rule as follows

$$\begin{aligned} f(x) &= 2x^3 - 33x^2 + 168x - 5 \\ &= 2x^3 - 33x^2 + 168x^1 - 5x^0 \\ f'(x) &= 3 \cdot 2x^{3-1} - 2 \cdot 33x^{2-1} + 1 \cdot 168x^{1-1} - 0 \cdot 5x^{0-1} \\ &= 6x^2 - 66x + 168 \end{aligned}$$

We can find the zeros/roots of $f'(x)$ using the quadratic formula or factoring $f'(x) = 6(x-4)(x-7)$ to determine the critical numbers x=4 and x=7.

**4** ??

**5** ??

**6** ??

**7** ??

**8** ??

**9** ??

**10** ??

**11** ??

**12** ??