

4. Solve the equation for $\ln\left(\frac{2}{x+1}\right) = 1$. Give your answer in the exact value (exponential form).
5. Given $f(x) = x - e^{0.5x}$, find the **coordinates** of any relative maximum or relative minimum points. Provide the exact values for the coordinates and a 3-decimal place approximation to these values.
6. Find the **coordinates** of a point on the graph of the function $f(x) = 1 - e^{3x}$ where the slope of graph is equal to -6 . Provide the exact values for the coordinates and a 3-decimal place approximation to these values.

Answer Key

1 $x = \pm 2$

2 $x = e^{1/2} = \sqrt{e}$

3 $x = -\ln(3) + 2$

4 $x = \frac{2}{e} - 1$

5 $x = 2\ln(2)$; $y = 2\ln(2) - 2$; maximum at $(1.386, -0.614)$

6 $x = \frac{\ln(2)}{3}$; $y = -1$; $(0.231, -1)$

Detailed Solutions

1 We first exponentiate both sides to remove x from the logarithm

$$\ln(x^2 - 3) = 0 \longrightarrow e^{\ln(x^2-3)} = x^2 - 3 = e^0 = 1$$

and then perform the following manipulations

$$x^2 - 3 = 1 \longrightarrow x^2 = 4 \longrightarrow x = \pm 2$$

2 We perform the following algebraic manipulations exponentiating to bring x out of the logarithm

$$2\ln(x) + 2 = 3 \longrightarrow 2\ln(x) = 1 \longrightarrow \ln(x) = \frac{1}{2} \longrightarrow x = e^{\frac{1}{2}} = \sqrt{e}$$

3 We first combine the terms on the right-hand side using properties of exponents

$$e^{2x} e^{2-3x} = e^{2x+2-3x} = e^{-x+2} = 3$$

we then take the logarithm of both sides and perform a few more manipulations

$$e^{-x+2} = 3 \longrightarrow \ln(e^{-x+2}) = -x + 2 = \ln(3) \longrightarrow x = -\ln(3 + 2)$$

4 We first exponentiate both sides

$$\ln\left(\frac{2}{x+1}\right) = 1 \longrightarrow e^{\ln\left(\frac{2}{x+1}\right)} = \left(\frac{2}{x+1}\right) = e^1 = e$$

and then perform the following algebraic manipulations

$$\left(\frac{2}{x+1}\right) = e \longrightarrow 2 = ex + e \longrightarrow x = \frac{2}{e} - 1$$

5 We begin by evaluating the derivative

$$f'(x) = \frac{d}{dx} [x - e^{0.5x}] = 1 - 0.5e^{0.5x}$$

and then looking for critical values

$$1 - 0.5e^{0.5x} = 0 \longrightarrow 2 = e^{0.5x} \longrightarrow \ln(2) = \ln(e^{0.5x}) = 0.5x \longrightarrow x = 2\ln(2) \approx 1.386$$

Now we find the y -coordinate

$$f(2 \cdot \ln(2)) = 2\ln(2) - e^{0.5 \cdot 2\ln(2)} = 2\ln(2) - e^{\ln(2)} = 2\ln(2) - 2 \approx -0.614$$

so we can express this critical point as $(1.386, -0.614)$. Let's use the first derivative test to determine if it's a maximum or minimum: $f'(1) = 0.176$ and $f'(2) = -0.359$ so it is an **absolute maximum**.

6 We first evaluate the derivative

$$f'(x) = \frac{d}{dx} [1 - e^{3x}] = -3e^{3x}$$

and find where the derivative evaluates to -6

$$-3e^{3x} = -6 \longrightarrow e^{3x} = 2 \longrightarrow 3x = \ln(2) \longrightarrow x = \frac{\ln(2)}{3} \approx 0.231$$

The y -coordinate is then

$$f\left(\frac{\ln(2)}{3}\right) = 1 - e^{3\frac{\ln(2)}{3}} = 1 - e^{\ln(2)} = 1 - 2 = -1$$

so the slope is -6 at the point $(0.231, -1)$.