1. Solve for x: $4ln(x) - ln(x^3) = 2$

2. Solve for x and **simplify** your answer: $ln(x^3) - ln(4x) + 2 = 0$

3. Solve for x: $ln(x) + ln(x + 6) = ln(2x - 3)$

4. Solve for x: $ln(x^4) - 2ln(x) = 1$

5. Given the function $f(x) = ln\left(\frac{x+1}{x^2}\right)$, first use properties of logarithms to re-write the function, and then find $f'(x)$. Write your answer in the form of a single rational expression.

6. Given the function $f(x) = ln\left[x^3 \cdot (x+1)^2\right]$, first use properties of logarithms to re-write the function, and then find $f'(x)$. Write your answer in the form of a single rational expression.

Answer Key

1 $x = e^2$

2 $x = \frac{2}{e}$ (Why is $x = -\frac{2}{e}$ not a solution?)

3 No solution.

4 $x = \sqrt{e}$

5 $f'(x) = \frac{-x-2}{x(x+1)}$ or $f'(x) = \frac{-x-2}{x^2+x}$

6 $f'(x) = \frac{5x+3}{x(x+1)}$ or $f'(x) = \frac{5x+3}{x^2+x}$

Detailed Solutions

1 We first use the logarithm power rule $ln(x^a) = a(x)$ with the goal of combining the logarithms using the quotient rule $ln(a) - ln(b) = ln(\frac{a}{b})$

$$4ln(x) - ln(x^3) = ln(x^4) - ln(x^3) = ln(\frac{x^4}{x^3}) = ln(x) = 2$$

from here we can exponentiate both sides to determine $x = e^2$.

2 We begin by using the logarithm quotient rule to combine the logarithms

$$ln(x^3) - ln(4x) + 2 = ln(\frac{x^3}{4x}) + 2 = ln(\frac{x^2}{4}) + 2 = 0$$

then use rearrange and exponentiate

$$ln(\frac{x^2}{4}) + 2 = 0 \longrightarrow \frac{x^2}{4} = e^{-2} \longrightarrow x^2 = 4e^{-2} \longrightarrow x = \pm\sqrt{\frac{4}{e^2}} = \pm\frac{2}{e}$$

but we cannot plug $x = -\frac{2}{e}$ into our original equation since logarithms cannot take negative values as inputs. The only solution is thus $x = \frac{2}{e}$.

3 We first combine the right side using the logarithm product rule $ln(a) + ln(b) = ln(a \cdot b)$

$$ln(x) + ln(x + 6) = ln(x(x + 6)) = ln(x^2 + 6x) = ln(2x - 3)$$

we then exponentiate both sides

$$ln(x^2 + 6x) = ln(2x - 3) \longrightarrow x^2 + 6x = 2x - 3 \longrightarrow x^2 + 4x + 3 = (x + 1)(x + 3) = 0$$

The factored equation is satisfied by $x = -1$ and $x = -3$. If we input these values into the right side of our original equation: $ln(2(-1) - 3) = ln(-5)$ and $ln(2(-3) - 3) = ln(-9)$ we can see they are undefined since logarithms cannot take negative values as input. Therefore there is no solution.

4 We first use the logarithm power rule $ln(x^a) = a(x)$ with the goal of combining the logarithms using the quotient rule $ln(a) - ln(b) = ln(\frac{a}{b})$

$$ln(x^4) - 2ln(x) = ln(x^4) - ln(x^2) = ln(\frac{x^4}{x^2}) = ln(x^2) = 1$$

we then exponentiate both sides

$$ln(x^2) = 1 \longrightarrow x^2 = e \longrightarrow x = \pm\sqrt{e}$$

Since logarithms cannot take on negative values the only solution is $x = \sqrt{e}$

5 Let us break up our equation using logarithm rules

$$f(x) = ln\left(\frac{x + 1}{x^2}\right) = ln(x + 1) - ln(x^2) = ln(x + 1) - 2ln(x)$$

We then take the derivative and combine everything into a single rational expression

$$f'(x) = \frac{d}{dx}\left[ln(x+1) - 2ln(x)\right] = \frac{1}{x+1} - 2\frac{1}{x} = \frac{x}{x(x+1)} - 2\frac{x+1}{x(x+1)} = \frac{-x-2}{x(x+1)}$$

6 Let us break up our equation using logarithm rules

$$f(x) = ln\left[x^3 \cdot (x+1)^2\right] = ln(x^3) + ln((x+1)^2) = 3ln(x) + 2ln(x+1)$$

then take the derivative and combine everything into a single rational expression

$$f'(x) = \frac{d}{dx}\left[3ln(x) + 2ln(x+1)\right] = 3\frac{1}{x} + 2\frac{1}{x+1} = 3\frac{x+1}{x(x+1)} + 2\frac{x}{x(x+1)} = \frac{5x+3}{x(x+1)}$$