1. Bacteria cells are being observed in order to determine a model for the number of cells, N, present at a point in time t hours after 50 cells are placed in a culture. The model to be used is an **exponential growth model** given by $N(t) = N_0 e^{kt}$; $t \geq 0$. After two hours, the number of cells is found to be 70.

   (a) Determine the function $N(t)$ that would be appropriate. Round the growth constant k to 2 decimal places.

   (b) At what rate is the number of cells changing at time $t = 3$ hours? Is the number of cells increasing or decreasing at this time? Round the rate to one decimal place, and provide the units.

   (c) At what point in time will the growth rate reach a value of 68 cells per hour? Round your answer to one decimal place.

2. A manufacturer has purchased a new piece of equipment for \$40,000. For accounting purposes, the value, V, of the machine at time t years from the present will be determined by the function $V(t) = 40000e^{-.03t}$; $t \geq 0$.

   (a) The company will keep the equipment until its value becomes \$30,000. According to this model, how long will the company keep the equipment? Round your answer to 1 decimal place.

   (b) At what rate is the equipment losing value at time $t = 3$ years after it was purchased? Round the rate to one decimal place, and provide the units.

   (c) Another piece of equipment purchased at the same time is valued according to the model $V_1(t) = 50000e^{-.02t}$; $t \geq 0$. Which piece of equipment is losing value at a faster rate at time $t = 5$ years? Round the rates to one decimal place, and provide the units.

3. **Atmospheric pressure** is the force per unit area exerted on a surface by the weight of **air** in the Earth's **atmosphere** above that surface. As elevation increases, there is less overlying atmospheric mass, so atmospheric pressure decreases with increasing elevation. In addition to less overlying mass, as you move to higher elevations the atmosphere is becoming less dense. That is, for a fixed volume of air, say one cubic foot, there are fewer air molecules at higher elevations. The atmospheric pressure (or barometric pressure), P, at a height of h miles above the earth's surface may be approximated by the following model: $\boldsymbol{P(h) = 14.7e^{-0.19h}}$ pounds per square inch; $h \geq 0$

   (a) At what altitude will the atmospheric pressure be equal to a value that is one-half of the pressure on the earth's surface? Find the answer in miles, h, then convert to a number of feet using 5280 feet per mile.

   (b) Determine the rate at which the pressure is decreasing at the altitude found in part (a).

   (c) The summit of Mt. Everest is at an altitude of 29,029 feet ($\approx$ 5.5 miles). **(1)** Determine the atmospheric pressure at the summit, and then express the pressure as a percentage of the pressure at the earth's surface. Is it clear why breathing is difficult at this altitude? **(2)** At what rate is the pressure decreasing at this altitude? Compare this value to the rate determined in part (b).

Answer Key

1 **(a)** $N(t) = 50e^{0.17t}$ **(b)** $N'(3) \approx 14.2$ cells per hour; increasing **(c)** $t \approx 12.2$ hours

2 **(a)** $t \approx 9.6$ years **(b)** \$1096.7 per year **(c)** $V'(5) \approx$ -\$1032.8 per year

3 **(a)** $h \approx 3.648$ miles ≈ 19262 ft. **(b)** $P'(3.648) \approx -1.40$ lbs per sq.inch per mile (or ≈ -0.000265 lbs per sq.inch per foot). **(c)** (1) $P(5.5) \approx 5.17$ lbs per sq.inch; $\frac{5.17}{14.7} \approx 0.352$ or 35.2% (a little more than $\frac{1}{3}$) (2) $P'(5.5) \approx -0.98$ lbs per sq.inch per mile (or ≈ -0.000186 lbs per sq.inch per foot); a **slower** rate of decrease compared to an altitude of 3.648 miles.

Detailed Solutions

1 (a) We know that at $t = 0$ there are 50 cells present, which we can plug into our model to determine

$$N(0) = 50 = N_0 e^{k\cdot 0} = N_0 e^0 = N_0$$

to determine $N_0 = 50$. We also know that after two hours $t = 2$ there are 70 cells present

$$N(2) = 70 = 50e^{k\cdot 2} \longrightarrow \frac{70}{50} = e^{k\cdot 2} \longrightarrow ln(\frac{7}{5}) = k \cdot 2$$

which simplifies further to $k = \frac{1}{2}ln(\frac{7}{5}) \approx 0.17$, and we conclude our growth function is given by $N(t) = 50e^{0.17t}$.

(b) We first evaluate the derivative

$$N'(t) = \frac{d}{dt}\left[50e^{0.17t}\right] = 50 \cdot 0.17e^{0.17t} = 8.5e^{0.17t}$$

then determine the rate of change at $t = 3$ hours via $N'(3) = 8.5e^{0.17\cdot 3} \approx 14.2$ cells per hour. Since the rate of change is positive the number is increasing.

(c) We want to find when the growth rate reaches 68 cells per hour, so we set the derivative to 68 and solve for t

$$68 = 8.5e^{0.17t} \longrightarrow 8 = e^{0.17t} \longrightarrow ln(8) = 0.17t \longrightarrow \frac{1}{0.17}ln(8) = t$$

Therefore the growth rate will be 68 cells per hour at $t = \frac{1}{0.17}ln(8) \approx 12.2$ hours.

2 (a) We use $V(t)$ to find how many years t until the value reaches \$30,000. We need t that satisfies the equation

$$30000 = 40000e^{-0.03t} \longrightarrow \frac{3}{4} = e^{-0.03t} \longrightarrow ln(\frac{3}{4}) = -0.03t \longrightarrow t = -\frac{1}{0.03}ln(\frac{3}{4})$$

to conclude $t = -\frac{1}{0.3}ln(\frac{3}{4}) \approx 9.6$ years.

(b) We first evaluate the derivative

$$V'(x) = \frac{d}{dt}\left[40000e^{-0.03t}\right] = 40000 \cdot (-0.03)e^{-0.03t} = -1200e^{-0.03t} \text{ per year}$$

and find that after $t = 3$ years the equipment is losing value at a rate of $V'(3) = -1200e^{-0.03\cdot 3} \approx -\$1096.7$ per year.

(c) We need to evaluate the derivative of the newly provided value equation

$$V_1'(t) = \frac{d}{dt}\left[50000e^{-.02t}\right] = 50000 \cdot (-.02)e^{-.02t} = 1000e^{-.02t}$$

Now let us find which is losing value faster after $t = 5$ years: $V'(5) = 1032.8$ and $V_1'(5) = -904.8$. Therefore $V'(t)$ is losing value faster after $t = 5$ years.

3 (a) To find when the pressure is half that of the surface pressure we need to know the surface pressure which is $P(0) = 14.7e^{-0.19\cdot 0} = 14.7$ pounds per square inch. Half of this is 7.35

pounds per square inch. To find the height where this occurs we use the given equation

$$7.35 = 14.7e^{-0.19h} \longrightarrow 0.5 = e^{-0.19h} \longrightarrow ln(0.5) = -0.19h \longrightarrow h = -\frac{1}{0.19}ln(0.5)$$

so the pressure is half that on Earth $h = -\frac{1}{0.19}ln(0.5) \approx 3.648$ miles. We then convert this into feet $h = 3.648 \text{ miles} \left(\frac{5280 \text{ feet}}{\text{mile}}\right) \approx 19261$ feet.

(b) To find the rate of pressure increase we evaluate the derivative

$$P'(h) = \frac{d}{dh}\left[14.7e^{-0.19h}\right] = 14.7(-0.19)e^{-0.19h} = 2.793e^{-0.19h}$$

and plug in the altitude $h = 3.648$ miles where the pressure is half that at the surface: $P'(3.648) = 2.793e^{-0.19\cdot(3.648)} \approx -1.40$ lbs per sq. inch per mile. We can also convert this answer into feet -0.000265 lbs per sq.inch per foot.

(c) (1) The atmospheric pressure at the summit is $P(5.5) = 14.7e^{-0.19\cdot 5.5} \approx 5.17$. As a percentage with respect to surface pressure this is $\frac{P(5.5)}{P(0)} = \frac{5.17}{14.7} \approx 35.2\%$. This pressure is roughly a third of that on the surface, so breathing at the summit of Everest is harder as the air is less dense. (2) To determine the rate at which the pressure is decreasing at the summit of Everest we evaluate $P'(5.5) = 2.793e^{-0.19\cdot 5.5} \approx -0.98$ lbs per sq. inch per mile. We can express this in feet as -0.000186 lbs per sq.inch per foot. The pressure at the summit is dropping slower than the altitude where the pressure was half that of the surface.