1. Mary invests \$30,000 in an account that pays 3% annual interest compounded continuously. Tom invests \$21,500 in an account that pays 4% annual interest compounded continuously.

   (a) Which account is growing at a faster rate at time $t = 0$? Provide the units on the rates of growth.

   (b) At what time are the amounts in the accounts growing at the same rate? Round your answer to one decimal place. Then determine the rates of growth at this time (rounded).

   (c) Which account is larger at the point in time found in part (b)?

2. Your parents are planning to retire in 8 years and they want to provide you with an "inheritance" while they are still alive. They have offered to give you \$75,000 when they retire ($t = 8$ years), or they will place an undetermined amount of money in an account immediately ($t = 0$ years) that you may withdraw in 8 years.

   (a) If they can put the money in an account for which the interest is compounded continuously at an annual rate of 4.5%, what is the minimum amount of money that you should request they invest now so that the amount you would receive in 8 years is at least \$75,000? Use the concept of present value to determine a solution. Round to the nearest dollar.

   (b) A week later, your parents decide to retire in 6 years, and they are still offering to give you \$75,000 at the time of their retirement ($t = 6$ years). However, they are not willing to place an amount of money in an account that is larger than the amount you determined in part (a). What would need to change with the account in order for the amount from part (a) to be sufficient to result \$75,000 in 6 years?

   (c) Find the value of the parameter you identified in part (b) that gives the desired results. Round to 3 decimal places.

Answer Key

1 **(a)** $A'_M(0) = \$900$ per year; $A'_T(0) = \$860$ per year **(b)** $t \approx 4.5$ years; $A'(4.5) \approx \$1030$ per year
(c) $A_M(4.5) \approx \$34336.1$; $A_T \approx \$25740.2$

2 **(a)** \$52326 **(b)** interest rate must increase **(c)** $r \approx 0.060$ or 6%

Detailed Solutions

1 Let us designate the dollar amount of Mary and Tom's accounts as $A_M(t)$ and $A_T(t)$, respectively. Since the interest is annual t is measured in years. Since it is compounded continuously both functions take the form A_0e^{kt}. Using the given information we can determine $A_M(t) = \$30000e^{0.03t}$ and $A_T(t) = \$21500e^{0.04t}$.

(a) To find which account is growing at a faster initial rate, we calculate the derivatives

$$A'_M(t) = \frac{d}{dt}\left[30000e^{0.03t}\right] = 30000(0.03)e^{0.03t} = \$900e^{0.03t} \text{ per year}$$

$$A'_T(t) = \frac{d}{dt}\left[21500e^{0.04t}\right] = 21500(0.04)e^{0.04t} = \$860e^{0.04t} \text{ per year}$$

and evaluate these at $t = 0$: $A'_M(0) = \$900$ per year and $A'_T(0) = \$860$ per year. We can see that Tom's account is growing at a faster rate at $t = 0$.

(b) The accounts are growing at the same rate when their derivatives are equal

$$900e^{0.03t} = 860e^{0.04t} \longrightarrow \frac{e^{0.04t}}{e^{0.03t}} = e^{0.04t-0.03t} = e^{0.01t} = \frac{900}{860} \longrightarrow 0.01t = ln(\frac{90}{86}) \longrightarrow t = \frac{1}{0.01}ln(\frac{90}{86})$$

we conclude that the accounts are growing at the same rate when $t = \frac{1}{0.01}ln(\frac{90}{86}) \approx 4.5$ years. Let's determine the exact rates $A'_M(4.5) = A'_T(4.5) = 900e^{0.03\cdot 4.5} \approx \$1030$ per year.

(c) We evaluate the value functions for $t = 4.5$ returning $A_M(4.5) = \$30000e^{0.03\cdot 4.5} \approx \$34336.1$ and $A_T(4.5) = \$21500e^{0.04\cdot 4.5} \approx \$25740.2$ and we can see that A_M is larger when $t = 4.5$ years.

2 (a) We want to find the present value of \$75,000 obtained through 8 years of continuously compounded interest at 4.5%. This amount is how much they would have to invest to match the 8 year offer. We can use the relationship between the present and future values $P = Fe^{-rt}$

$$P = 75000e^{-0.045\cdot 8} \approx \$52326$$

(b) The parents are not willing to put more than \$ 52,326 into an account, and the time $t = 6$ is fixed, so the only thing that can change is the annual interest rate r.

(c) We want to find r such that \$52,326 grows to \$75,000 over 6 years via

$$52326e^{r\cdot 6} = 75000 \longrightarrow e^{r\cdot 6} = \frac{75000}{52326} \longrightarrow r\cdot 6 = ln\left(\frac{75000}{52326}\right) \longrightarrow r = \frac{1}{6}ln\left(\frac{75000}{52326}\right) \approx 0.060$$

Therefore we can only reach the 6 year offer with an interest rate of 6%.