1. Evaluate $\int \left(-\frac{2}{x^4} + 3x \right) dx$.

2. Evaluate $\int \left(2e^{\frac{1}{3}x} - 2 \right) dx$.

3. Evaluate $\int \left(\frac{3}{x} + 3x^2 \right) dx$.

4. Evaluate $\int \left(3\sqrt{x} - \frac{1}{2}e^{-x} \right) dx$.

5. The **rate** at which the population of a city is changing at time t months from the present is given by $P'(t) = 100e^{0.02t}$ people per month. If the present population is 6000 people, that is, $P(0) = 6000$, determine the function $P(t)$.

6. A manufacturer of antifreeze has determined that when they are producing x barrels per day, the marginal cost may be estimated by $C'(x) = 0.3\sqrt{x} + 0.01x - 0.01$ hundreds of dollars per barrel. The fixed cost of the operation, that is when the production level is $x = 0$ barrels, is $C(0) = 1$ (hundred dollars). Determine the daily cost function $C(x)$.

Answer Key

1 $\dfrac{2}{3x^3} + \dfrac{3}{2}x^2 + C$

2 $6e^{\frac{1}{3}x} - 2x + C$

3 $3\ln|x| + x^3 + C$

4 $2x^{\frac{3}{2}x} + \frac{1}{2}e^{-x} + C$

5 $P(t) = 5000e^{0.02t} + 1000$

6 $C(x) = 0.2x^{\frac{3}{2}} + 0.005x^2 - 0.01x + 1$

Detailed Solutions

1 We can integrate each term separately since they are being summed

$$\int \left(-\frac{2}{x^4} + 3x\right) dx = \int \left(-\frac{2}{x^4}\right) dx + \int (3x)\, dx$$

and then use the power rule for integration $\int x^m dx = \frac{x^{m+1}}{m+1}$ noting $\frac{1}{x^4} = x^{-4}$

$$\int \left(-2x^{-4}\right) dx + \int \left(3x^{1}\right) dx = \frac{-2}{-4+1}x^{-4+1} + \frac{3}{1+1}x^{1+1} + C = \frac{2}{3x^3} + \frac{3}{2}x^2 + C$$

We need the $+C$ since we are evaluating an indefinite integral.

2 We can integrate each term separately since they are being summed

$$\int \left(2e^{\frac{1}{3}x} - 2\right) dx = \int \left(2e^{\frac{1}{3}x}\right) dx + \int (-2)\, dx$$

integrating a constant a returns ax and we use the following rule for exponentials $\int e^{ax} dx = \frac{1}{a}e^{ax}$

$$\int \left(2e^{\frac{1}{3}x}\right) dx + \int (-2)\, dx = \frac{2}{1/3}e^{\frac{1}{3}x} - 2x + C = 6e^{\frac{1}{3}x} - 2x + C$$

We need the $+C$ since we are evaluating an indefinite integral.

3 We can integrate each term separately since they are being summed

$$\int \left(\frac{3}{x} + 3x^2\right) dx = \int \left(\frac{3}{x}\right) dx + \int \left(3x^2\right) dx$$

It is important to note that the power rule for integration fails for $\frac{1}{x}$, and instead $\int \frac{1}{x} dx = ln|x|$ since the derivative of $ln(x)$ is $\frac{1}{x}$. The straight bars indicate absolute value. If we use the power rule we would have to divide by 0.

$$\int \left(\frac{3}{x}\right) dx + \int \left(3x^2\right) dx = 3ln|x| + \frac{3}{2+1}x^{2+1} + C = 3ln|x| + x^2 + C$$

We need the $+C$ since we are evaluating an indefinite integral.

4 We can integrate each term separately since they are being summed

$$\int \left(3\sqrt{x} - \frac{1}{2}e^{-x}\right) dx = \int \left(3\sqrt{x}\right) dx + \int \left(-\frac{1}{2}e^{-x}\right) dx$$

We can use the power rule for integrals noting $\sqrt{x} = x^{\frac{1}{2}}$ and $\int e^{ax} dx = \frac{1}{a}e^{ax}$

$$\int \left(3x^{\frac{1}{2}}\right) dx + \int \left(-\frac{1}{2}e^{-1x}\right) dx = \frac{3}{\frac{1}{2}+1}x^{\frac{1}{2}+1} - \frac{1}{2}\left(\frac{1}{-1}\right) e^{-1}x + C = 2x^{\frac{3}{2}x} + \frac{1}{2}e^{-x} + C$$

We need the $+C$ since we are evaluating an indefinite integral.

5 To get from $P'(t)$ to $P(t)$ we integrate

$$P(t) = \int P'(t)dt = \int 100e^{0.02t}dt = \frac{100}{0.02}e^{0.02t} + C = 5000e^{0.02t} + C$$

and we can determine the unknown C from the condition $P(0) = 6000$ via

$$P(0) = 6000 = 5000e^{0.02\cdot 0} + C = 5000 + C \longrightarrow 6000 = 5000 + C \longrightarrow C = 1000$$

Therefore we can express the population $P(t) = 5000e^{0.02t} + 1000$.

6 We find $C(x)$ by integrating $C'(x)$

$$C(x) = \int C'(x)dx = \int \left(0.3\sqrt{x} + 0.01x - 0.01\right) dx = \frac{0.3}{\frac{3}{2}x^{\frac{3}{2}}} + \frac{0.01}{2}x - 0.01x + A$$

which simplifies further to $C(x) = 0.2x^{\frac{3}{2}} + 0.005x^2 - 0.01x + A$. Note that the unknown constant is designated as A to avoid confusion with the cost function $C(x)$. We determine the unknown constant A via the condition $C(0) = 1$ hundred dollars

$$C(0) = 1 = 0.2(0)^{\frac{3}{2}} + 0.005(0)^2 - 0.01(0) + A = 0 + A \longrightarrow A = 1$$

so we conclude $C(x) = 0.2x^{\frac{3}{2}} + 0.005x^2 - 0.01x + 1$ hundreds of dollars.