1. Evaluate $\int_1^e \left(\frac{2}{x}\right) dx$.

2. Evaluate $\int_{-1}^1 \left(x^3\right) dx$.

3. Evaluate $\int_{-1}^1 \left(3x^2 - 3\right) dx$.

4. Evaluate $\int_{-1}^0 (e^x - 1)\, dx$. Give the exact answer, then provide a 3-decimal place approximation.

5. The **rate** at which the population of a city is changing at time t months from the present is given by $P'(t) = 100e^{0.02t}$ people per month. Find **net change** in the population size over the period from time $t = 2$ months to $t = 6$ months. Indicate if the population has increased or decreased over the period. Round to the nearest whole number.

6. A manufacturer of antifreeze has determined that when they are producing x barrels per day, the marginal cost may be estimated by $C'(x) = 0.3\sqrt{x} + 0.01x - 0.01$ hundreds of dollars per barrel. Find the **net change** in the total production cost if the production level is increased from $x = 80$ barrels per day to $x = 100$ barrels per day. Round your answer to one decimal place.

Answer Key

1 2

2 0

3 -4

4 $-\dfrac{1}{e} \approx -0.368$

5 Increase of 433 people

6 74.7 hundreds of dollars or \$7470.

Detailed Solutions

1 We evaluate this integral knowing that $\frac{d}{dx}ln(x) = \frac{1}{x}$ and evaluating the result at the endpoints

$$\int_1^e \left(\frac{2}{x}\right) dx = 2ln(x)\Big|_1^e = 2ln(e) - 2ln(1) = 2 \cdot 1 - 2 \cdot 0 = 2$$

There is no constant of integration when we perform *definite* integrals.

2 We evaluate this integral using the power rule for integration $\int x^m dx = \frac{x^{m+1}}{m+1}$

$$\int_{-1}^1 (x^3)\, dx = \frac{x^4}{4}\Big|_{-1}^1 = \frac{(1)^4}{4} - \frac{(-1)^4}{4} = \frac{1}{4} - \frac{1}{4} = 0$$

We can see why this integral evaluates to zero by noting that x^3 is an *odd*.

3 We use the power rule for integration

$$\int_{-1}^1 (3x^2 - 3)\, dx = \left(3\frac{x^3}{3} - 3x\right)\Big|_{-1}^1 = [1^3 - 3(1)] - [(-1)^3 - 3(-1)] = [-2] - [2] = -4$$

4 We use both the power rule and integration on exponential functions

$$\int_{-1}^0 (e^x - 1)\, dx = (e^x - x)\Big|_{-1}^0 = [e^0 - 0] - [e^{-1} - (-1)] = [1] - [\frac{1}{e} + 1] = -\frac{1}{e} \approx -0.368$$

5 The net change from 2 months to 6 months given the population rate of change $P'(t)$ is $\int_2^6 P'(t)dt$. We then use the integration rule for exponentials

$$\int_2^6 P'(t)dt = \int_2^6 100e^{0.02t}dt = \left(\frac{100}{0.02}e^{0.02t}\right)\Big|_2^6 = \left(5000e^{0.02t}\right)\Big|_2^6 = 5000(e^{0.02\cdot 6} - e^{0.02\cdot 2}) \approx 433$$

So we can conclude that there was a net increase of 433 people.

6 If we want to find the net change from $x = 80$ barrels per day to $x = 100$ barrels per day given the marginal cost $C'(x)$ is $\int_{80}^{100} C'(x)dx$. We also express $\sqrt{x} = x^{\frac{1}{2}}$

$$\int_{80}^{100} C'(x)dx = \int_{80}^{100} \left(0.3x^{\frac{1}{2}} + 0.01x - 0.01\right) dx = \left(0.3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 0.01\frac{x^2}{2} - 0.01x\right)\Big|_{80}^{100}$$
$$= \left(0.2x^{\frac{3}{2}} + 0.005x^2 - 0.01x\right)\Big|_{80}^{100} \approx 74.7$$

Since $C'(x)$ measures hundred of dollars per barrel and we have integrated over the number of barrels, $C(x)$ measures hundreds of dollars. Therefore the change in production cost is 74.7 hundreds of dollars or $7470.