1. Evaluate $\displaystyle\int \left(\frac{[\ln(x)]^2}{x}\right) dx$. *Note: Re-write the integrand as* $\displaystyle\frac{1}{x} \cdot [\ln(x)]^2$.

2. Evaluate $\displaystyle\int \left(12xe^{-3x^2}\right) dx$.

3. Evaluate $\displaystyle\int \left(\frac{2xe^{x^2}}{1+e^{x^2}}\right) dx$. Let $u = 1 + e^{x^2}$

4. Evaluate $\displaystyle\int \left(\frac{6x^2}{x^3+3}\right) dx$.

5. Evaluate $\int \left(x^2\sqrt{x^3 - 10}\right) dx$. *Note: First convert the radical to exponential form.*

6. Evaluate $\int \left(\frac{x}{e^{x^2}}\right) dx$. *Note: Recall that* $\frac{1}{e^a} = e^{-a}$

Answer Key

1 $F(x) = \frac{1}{3} \cdot [\ln(x)]^3 + C$

2 $F(x) = -2e^{-3x^2} + C$

3 $F(x) = \ln(1 + e^{x^2}) + C$

4 $F(x) = 2\ln(x^3 + 3) + C$

5 $F(x) = \frac{2}{9}(x^3 - 10)^{\frac{3}{2}} + C$

6 $F(x) = -\frac{1}{2}e^{-x^2} + C$

Detailed Solutions

1 We want to identify elements of the integrand that show up alongside their derivative. Define $u = ln(x)$ then $du = \frac{1}{x}dx$ and our integral can be written

$$\int \left(\frac{[\ln(x)]^2}{x}\right) dx = \int u^2 du = \frac{u^3}{3} + C = \frac{1}{3} \cdot [\ln(x)]^3 + C$$

2 We first observe that e^{-3x^2} appears in the integrand, and we can use the chain rule to evaluate $\frac{d}{dx}e^{-3x^2} = -6xe^{-3x^2}$. We can use this to solve the integral. Alternatively we can define $u = x^2$ and $du = 2xdx$ to rewrite the integral

$$\int \left(12xe^{-3x^2}\right) dx = \int \left(6 \cdot 2xe^{-3x^2}\right) dx = \int \left(6 \cdot e^{-3u}\right) du = 6\frac{1}{-3}e^{-3u} + C = -2e^{-3x^2} + C$$

3 We use the problem hint and define $u = 1 + e^{x^2}$ and determine $du = 2xe^{x^2}dx$ and our integral becomes

$$\int \left(\frac{2xe^{x^2}}{1 + e^{x^2}}\right) dx = \int \left(\frac{1}{u}\right) du = ln(u) + C = ln\left(1 + e^{x^2}\right) + C$$

4 We can identify x^2dx and x^3 in the integrand, one being the derivative of the other. We can define $u = x^3 + 3$ and find $du = 3x^2dx$ such that

$$\int \left(\frac{6x^2}{x^3 + 3}\right) dx = \int \left(\frac{2 \cdot 3x^2}{x^3 + 3}\right) dx = \int \left(\frac{2}{u}\right) du = 2ln(u) + C = 2ln(x^3 + 3) + C$$

5 Let's express $x^2\sqrt{x^3 - 10} = x^2(x^3 - 10)^{\frac{1}{2}}$. We then define $u = x^3 - 10$ and determine $du = 3x^2$ (or $\frac{1}{3}du = x^2$) to rewrite our integral and evaluate

$$\int x^2(x^3 - 10)^{\frac{1}{2}} dx = \int \frac{1}{3}(u)^{\frac{1}{2}} du = \frac{1}{3}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{9}u^{\frac{3}{2}} + C = \frac{2}{9}(x^3 - 10)^{\frac{3}{2}} + C$$

6 We have both xdx and x^2 appearing in our integrand, so we define $u = x^2$ and $du = 2xdx$ (or $\frac{1}{2} = xdx$) to rewrite and evaluate or integral

$$\int \left(\frac{x}{e^{x^2}}\right) dx = \int \frac{1}{2}\left(\frac{1}{e^u}\right) du = \int e^{-u} du = \frac{1}{-1}e^{-u} + C = -\frac{1}{2}e^{-x^2} + C$$