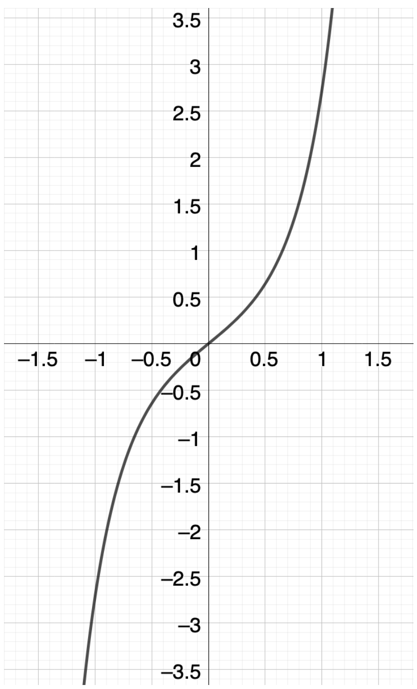
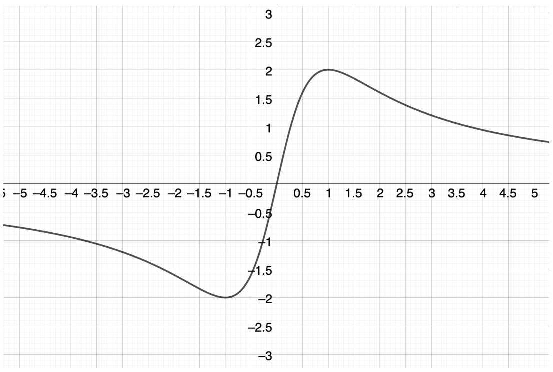
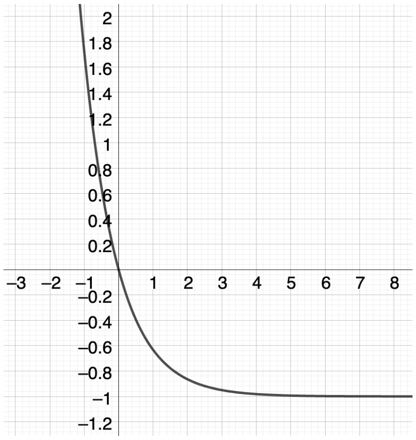
1. Given the function $f(x) = xe^{x^2}$ and its graph given below, answer the following questions:
   (a) Determine and shade the **(total) area** bounded by the $x-$axis and the graph of f on the interval from $x = -1$ to $x = 0$. Give the exact answer, and a 3-decimal place approximation.
   (b) Determine and shade the **(total) area** bounded by the $x-$axis and the graph of f on the interval from $x = 0$ to $x = 1$. Give the exact answer, and a 3-decimal place approximation.
   (c) Determine and shade the **(total) area** bounded by the $x-$axis and the graph of f on the interval from $x = -1$ to $x = 1$. Give the exact answer, and a 3-decimal place approximation.
   (d) Determine the **net signed area** bounded by the $x-$axis and the graph of f on the interval from $x = -1$ to $x = 1$.



2. Given the function $f(x) = \dfrac{4x}{x^2+1}$ and its graph given below, answer the following questions:
   (a) <u>Determine</u> and <u>shade</u> the **(total) area** <u>below</u> the x−axis on the interval from $x = -1$ to $x = 2$. Give the exact answer, and a 3-decimal place approximation.
   (b) <u>Determine</u> and <u>shade</u> the **(total) area** bounded by the x−axis and the graph of f on the interval from $x = -1$ to $x = 2$. Give the exact answer, and a 3-decimal place approximation.
   (c) Determine the **net signed area** bounded by the x−axis and the graph of f on the interval from $x = -1$ to $x = 2$. Give the exact answer, and a 3-decimal place approximation.



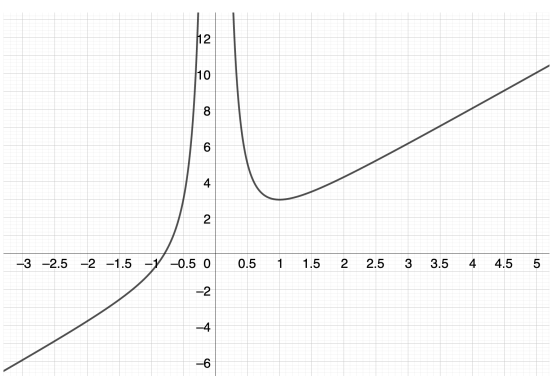
3. Determine and shade the (total) area bounded by the $x-$axis and the graph of the function $f(x) = e^{-x} - 1$ on the interval from $x = -1$ to $x = 1$. Give the exact form of the area, then a 3-decimal place approximation. The graph of f is given below.



4. Given the function $f(x) = \dfrac{1}{x^2} + 2x$ on the and its graph given below, answer the following questions:

   (a) Determine and shade the **(total) area** bounded by the $x-$axis and the graph of f on the interval from $x = 1$ to $x = 2$. Give the exact answer.

   (b) Determine the **net signed area** bounded by the $x-$axis and the graph of f on the interval from $x = 1$ to $x = 2$. Give the exact answer.



Answer Key

1 **(a)** $\int_{-1}^{0} |f(x)|\,dx = \int_{-1}^{0} -f(x)\,dx = -\int_{-1}^{0} f(x)\,dx = -\frac{1}{2}e^{x^2}\Big|_{-1}^{0} = \frac{1}{2}e - \frac{1}{2} \approx 0.859.$

(b) $\int_{0}^{1} |f(x)|\,dx = \int_{0}^{1} f(x)\,dx = \frac{1}{2}e^{x^2}\Big|_{0}^{1} = \frac{1}{2}e - \frac{1}{2} \approx 0.859.$

(c) total area $= \int_{-1}^{1} |f(x)|\,dx = \int_{-1}^{0} |f(x)|\,dx + \int_{0}^{1} |f(x)|\,dx$ =area below the $x-$axis + area above the $x-$axis= $e - 1 \approx 1.718$

(d) net signed area $= \int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx = -\left(\frac{1}{2}e - \frac{1}{2}\right) + \left(\frac{1}{2}e - \frac{1}{2}\right)$=area above the $x-$axis $-$ area below the $x-$axis= 0

2 **(a)** $\int_{-1}^{0} |f(x)|\,dx = \int_{-1}^{0} -f(x)\,dx = -\int_{-1}^{0} f(x)\,dx = -2ln|x^2+1|\big|_{-1}^{0} = ln(4) \approx 1.386.$

(b) total area $= \int_{-1}^{2} |f(x)|\,dx = \int_{-1}^{0} |f(x)|\,dx + \int_{0}^{2} |f(x)|\,dx$ =area below the $x-$axis + area above the $x-$axis= $ln(4) + ln(25) \approx 4.605$

(c) net signed area $= \int_{-1}^{2} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$=area above the $x-$axis $-$ area below the $x-$axis≈ 1.833

3 area above the $x-$axis= $-e^{-x} - x|_{-1}^{0} = -2 + e \approx 0.718$; total area= $-2 + e + e^{-1} \approx 1.086$

4 **(a)** total area $= -\frac{1}{x} + x^2\Big|_{1}^{2} = 3.5$ **(b)** net signed area $= 3.5$.

Detailed Solutions

1 Let us evaluate the following indefinite integral to make subsequent calculations easier using the u-substitution $u = x^2$ and $du = 2xdx$

$$\int xe^{x^2}dx = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

(a) From the graph we can see that the integral from $x = -1$ to $x = 0$ will be negative. We are looking for the total answer, so we need to flip the sign on our result

$$\int_{-1}^{0} xe^{x^2}dx = \frac{1}{2}e^{x^2}\Big|_{-1}^{0} = \frac{1}{2}\left(e^{0^2} - e^{-1^2}\right) = \frac{1}{2}(1-e) \approx -0.859$$

and the total/unsigned area is 0.859.

(b) From the graph we can see that the integral from $x = 0$ to $x = 1$ will be positive, so we need only evaluate

$$\int_{0}^{1} xe^{x^2}dx = \frac{1}{2}e^{x^2}\Big|_{0}^{1} = \frac{1}{2}\left(e^{1^2} - e^{0^2}\right) = \frac{1}{2}(e-1) \approx 0.859$$

(c) We want the total area from $x = -1$ to $x = 1$, which means we need to add the previous two answer as integrating might cause signed portions to cancel. This area is thus $0.859 + 0.859 =$ 1.718

(d) The net signed area is the integral from $x = -1$ to $x = 1$, so we evaluate

$$\int_{-1}^{1} xe^{x^2}dx = \frac{1}{2}e^{x^2}\Big|_{-1}^{1} = \frac{1}{2}\left(e^{1^2} - e^{-1^2}\right) = \frac{1}{2}(e-e) = 0$$

2 Let us evaluate the following indefinite integral to make subsequent calculations easier using the u-substitution $u = x^2$ and $du = 2xdx$

$$\int \frac{4x}{x^2+1}dx = 2\int \frac{du}{u+1} = 2ln(u+1) + C = 2ln(x^2+1) + C$$

(a) From the graph we can see that the integral from $x = -1$ to $x = 0$ will be negative. We are looking for the total answer, so we need to flip the sign on our result

$$\int_{-1}^{0} \frac{4x}{x^2+1}dx = 2ln(x^2+1)\Big|_{-1}^{0} = 2\left(ln(0^2+1) - ln(-1^2+1)\right) = 2(0 - ln(2)) = -2ln(2) \approx -1.386$$

and the total/unsigned area is 1.386.

(b) From the graph we can see that the integral from $x = 0$ to $x = 1$ will be positive, so we need only evaluate

$$\int_{0}^{1} \frac{4x}{x^2+1}dx = 2ln(x^2+1)\Big|_{0}^{1} = 2\left(ln(1^2+1) - ln(0^2+1)\right) = 2(ln(2) - 0) = 2ln(2) \approx 1.386$$

(c) The net signed area from $x = -1$ to $x = 2$ is the integral evaluated over those bounds

$$\int_{-1}^{2} \frac{4x}{x^2+1}dx = 2ln(x^2+1)\Big|_{-1}^{2} = 2\left(ln(2^2+1) - ln(-1^2+1)\right) = 2(ln(5) - ln(2)) \approx 1.833$$

3 To find the total area from $x = -1$ to $x = 1$ we need to break our integral into two parts since the area will possess a negative sign when we integrate past $x > 0$. We first evaluate the total area from $x = -1$ to $x = 0$

$$\int_{-1}^{0} \left(e^{-x} - 1\right) dx = -e^{-x} - x\Big|_{-1}^{0} = \left(-e^{-0} - 0\right) - \left(-e^{-(-1)} - (-1)\right) = -1 + e - 1 = e - 2$$

then find the total area from $x = 0$ to $x = 1$

$$\int_{0}^{1} \left(e^{-x} - 1\right) dx = -e^{-x} - x\Big|_{0}^{1} = \left(-e^{-1} - 1\right) - \left(-e^{-0} - 0\right) = -\frac{1}{e} - 1 + 1 = -\frac{1}{e}$$

but we need to flip the sign to arrive at $\frac{1}{e}$. Together we have $\frac{1}{e} + e - 2 \approx 1.086$.

4 (a) If we are looking for the total area from $x = 1$ to $x = 2$ we need only integrate since the area will be positive for the entire interval

$$\int_{1}^{2} \left(\frac{1}{x^2} + 2x\right) dx = \frac{-1}{x} + x^2\Big|_{1}^{2} = \left(\frac{-1}{2} + 2^2\right) - \left(\frac{-1}{1} + 1^2\right) = -\frac{1}{2} + 4 + 1 - 1 = 3.5$$

(b) To find the net signed area we need to evaluate the integral from $x = 1$ to $x = 2$, but this was already done with no modification in part (a) so the answer is 3.5.