1. The concentration, C, of a drug in a patient's bloodstream at a time t hours after injection is predicted using the function $C(t) = 10te^{-t^2}$ milligrams per liter. Find the average concentration over the intervals $[0, 0.5]$ and $[2.5, 3]$ and compare. Round answers to 2 decimal places.

2. Suppose you deposit \$4000 at 8% interest compounded continuously. Find the average value of your account during the first 2 years. Round your answer to 2 decimal places.

3. The temperature of a town (in degrees Fahrenheit) can be estimated by the function $f(x) = 0.025x^4 - 0.067x^3 + 5x^2 - 21x + 35$ where $x = 1$ represents January, $x = 2$ represents February, etc. Find the average temperature from March to June. Round your answer to 2 decimal places.

Answer Key

1 On $[0, 0.5]$ (1^{st} half-hour], $f_{ave} = -10e^{-t^2}\Big|_0^{0.5} \approx 2.21$ mg. per liter. On $[2.5, 3]$, $f_{ave} = -10e^{-t^2}\Big|_{2.5}^{3} \approx$ 0.02 mg. per liter. The concentration is very low after 2.5 hours. Check the graph of the function and see why.

2 $f_{ave} = \dfrac{1}{2} \cdot \left[50000e^{0.08x}\right]_0^2 \approx \$4337.77$.

3 $f_{ave} = \dfrac{1}{3} \cdot \left[0.005x^5 - 0.01675x^4 + 1.66667x^3 - 10.5x^2 + 35x\right]_3^6 \approx 51.27\ {}^\circ F.$

Detailed Solutions

1 We find the average value of a function over continuous domains by integrating over that domain and dividing by the length of the interval. Let's first evaluate the indefinite integral of $C(t) = 10te^{-t^2}$ using $u = -t^2$ and $du = -2tdt$ (or $-\frac{1}{2}du = tdt$)

$$\int C(t)dt = \int 10te^{-t^2}dt = \int 10(-\frac{1}{2})e^{u}du = -5\int e^{u}du = -5e^{u} = -5e^{-t^2}$$

We neglect the unknown constant since we are concerned with definite integrals. Then we can easily evaluate the averages over $[0, 0.5]$ and $[2.5, 3]$, both of which have length $\frac{1}{2}$

$$\frac{1}{1/2}\int_0^{0.5} C(t)dt = -10e^{-t^2}\Big|_0^{0.5} = -10e^{-(0.5)^2} - \left(-10e^{-0^2}\right) \approx 2.21 \text{ mg. per liter}$$
$$\frac{1}{1/2}\int_{2.5}^{3} C(t)dt = -10e^{-t^2}\Big|_{2.5}^{3} = -10e^{-(3)^2} - \left(-10e^{-(2.5)^2}\right) \approx 0.02 \text{ mg. per liter}$$

The average concentration is very low after 2.5 hours. Note that the average value has the same units as the original function even though we integrate. This occurs because we divide by the length of the interval which has the same units as dt.

2 We can express the value accumulated using the given information as $A(t) = 4000e^{0.08t}$ where t is measured in years. We then look for the average over 2 years

$$\frac{1}{2}\int_0^2 4000e^{0.08t}dt = 2000\left(\frac{1}{0.08}e^{0.08t}\right)\Big|_0^2 = 25000\left(e^{0.08(2)} - e^{0.08(0)}\right) \approx \$4337.77$$

3 To find the average from March to June we are looking from $x = 3$ to $x = 6$, so

$$\frac{1}{6-3}\int_3^6 \left[0.025x^4 - 0.067x^3 + 5x^2 - 21x + 35\right]dx = \frac{1}{3}\left[\frac{0.025}{5}x^5 - \frac{0.067}{4}x^4 + \frac{5}{3}x^3 - \frac{21}{2}x^2 + 35x\right]\Big|_3^6$$
$$= \frac{1}{3}\left[\frac{0.025}{5}6^5 - \frac{0.067}{4}6^4 + \frac{5}{3}6^3 - \frac{21}{2}6^2 + 35(6)\right] - \frac{1}{3}\left[\frac{0.025}{5}3^5 - \frac{0.067}{4}3^4 + \frac{5}{3}3^3 - \frac{21}{2}3^2 + 35(3)\right] \approx 51.27^\circ F$$