

- (c) Return to the scenario in part (a). Suppose you choose the \$1,000,000 now (which you should). The state is short on funds and was hoping you would have selected the payments of \$200,00 over 5 years. They respond by saying they are willing to extend the annual payment stream by one more year for a total of 6 years. Would this change your decision?

2. A business is forecasting that their profits will be increasing over the next 5 years. Their prediction is that the annual profit stream will be $50,000e^{.03t}$ dollars per year on a continuous basis (the profit stream is increasing over time). A venture capitalist has approached the business owner and wants to purchase the business. What is the minimum amount the owner should demand in order to get the same value as keeping the business for 5 years? Assume that invested money will earn interest at an annual rate of 4% compounded continuously.

3. An employee of a company that provides supplemental retirement programs is considering 2 options. For the 1st option, she will deposit \$20,000 per year on a continuous basis in an account that will pay interest at a guaranteed annual rate of 4.5% compounded continuously. She will begin this program at her current age of 40 (consider this to be time $t = 0$) and may retire at age 60 ($t = 20$). For the 2nd option, she will begin at the same age of 40 and deposit \$15,000 per year on a continuous basis in an account that will also pay interest at an annual rate of 4.5% compounded continuously. She would retire at age 65 ($t = 25$) under this option.
- (a) Which account will have a greater value at the time of her retirement specified in each option? Round your answers to the nearest dollar (**don't round until your final answer**).

- (b) She likes the future value of option 2 (see answer below), but likes the retirement time of option 1. Using option 1, what amount would she need to deposit annually on a continuous basis in order to have the future value of the account be equal to \$693,405 at the end of 20 years (this is future value of option 2)?

4. An investment group has made an opportunity available which would require you to provide \$100,000 immediately. In return, you are guaranteed a continuous stream of income of \$24,000 per year over the next 5 years. The money you would receive can be invested as it is received at an annual rate of 5% compounded continuously.
- (a) Determine the **present value** of the income stream described above. Round your answer to the nearest dollar.
- (b) Of course, another option is to put your \$100,000 in an account earning interest at an annual rate of 5% compounded continuously and leave it there for 5 years. Based on your result in part (a), would you give up your \$100,000 and make this investment? YES or NO, and JUSTIFY your choice.
5. A recent college graduate has found a job and wants to begin saving money for a down payment on a house that she would like to purchase in 5 years. She has found a fund that will guarantee an annual interest rate of 5% compounded continuously. Her contribution to the fund will be \$12,000 per year, which is assumed to be made on a continuous basis over time.
- (a) Determine the future value of her account at time $t = 5$ years from the present. Round your answers to the nearest dollar (**don't round until your final answer**).

- (b) Her goal was to have \$75,000 after 5 years for the down payment. What amount would she need to deposit annually on a continuous basis in order to have the future value of the account be equal to \$75,000 at the end of 5 years? Round your answer to the nearest whole dollar.

Answer Key

- 1 (a)** $PV = 200,000 \left[\frac{1}{-0.035} e^{-0.035t} \right]_0^5 \approx \$917,388$; you could invest this much in a savings account now and have the same value at the end of 5 years as the stream of payments. If you take \$1,000,000 now and invest it, you would have even more. Take the \$1,000,000 **(b)** $PV = 225,000 \left[\frac{1}{-0.035} e^{-0.035t} \right]_0^5 \approx \$1,032,062$. Take this option as they are only offering \$1,000,000 right now **(c)** $PV = 200,000 \left[\frac{1}{-0.035} e^{-0.035t} \right]_0^6 \approx \$1,082,376$ which exceeds the lump sum, so take this option.
- 2** $PV = 50,000 \left[\frac{1}{-0.01} e^{-0.01t} \right]_0^5 \approx \$243,853$. Any amount less than this would result in a value after 5 years that is less than the amount you would have keeping the business and income stream.
- 3 (a)** Option 1: $FV = 20,000 \left[\frac{1}{-0.045} e^{0.045(20-t)} \right]_0^{20} \approx \$648,712$; option 2: $FV = 15,000 \left[\frac{1}{-0.045} e^{0.045(25-t)} \right]_0^{25} \approx \$693,406$. The 2nd option gives greater value but requires working longer. **(b)** $693,405 = K \left[\frac{1}{-0.045} e^{0.045(20-t)} \right]_0^{20} \implies K \approx \$21,378$ per year. Note that she needs to invest more each year but has a 20-year timeframe until retirement.
- 4 (a)** $PV = 24,000 \left[\frac{1}{-0.05} e^{-0.05t} \right]_0^5 \approx \$106,176$. **(b)** Yes. If you had the income stream, the present value tells you that the minimum amount you would accept to release it is \$106,176, so its current value is more than you are being asked to invest, which makes it a good investment.
- 5 (a)** $FV = 12,000 \left[\frac{1}{-0.05} e^{0.05(5-t)} \right]_0^5 \approx \$68,166$. **(b)** $75,000 = K \left[\frac{1}{-0.05} e^{0.05(5-t)} \right]_0^5 \implies K \approx \$13,203$ annual deposit on a continuous basis.

Detailed Solutions

1 (a) We calculate the present value

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4 ??

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