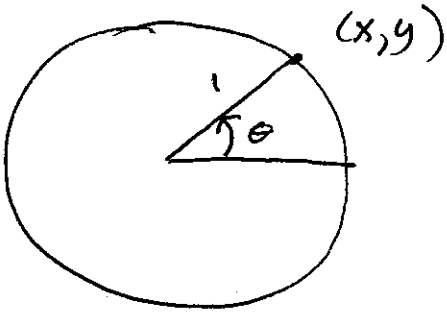
$$\lim_{x \to a} f(x) = f(a) \iff \lim_{h \to 0} f(a+h) = f(a)$$



$\sin\theta = y$

$\lim_{\theta \to 0} \sin\theta = 0$

So $\sin\theta$ is continuous at $\theta = 0$.

$\cos\theta = x$, $\lim_{\theta \to 0} \cos\theta = 1$

So $\cos\theta$ is continuous at $\theta = 0$.

Let $f(x) = \sin x$

$$\lim_{h \to 0} \sin(x+h) = \lim_{h \to 0} (\sin x \cos h + \cos x \sin h)$$

$$= \sin x \cdot \lim_{h \to 0} \cos h + \cos x \cdot \lim_{h \to 0} \sin h$$

$$= \sin x \cdot 1 + (\cos x) \cdot 0$$

$$= \sin x .$$

Hence f is continuous everywhere.