Math 4200

1. Show that $sin\, t$ and $cos\, t$ are continuous functions.

2. Show that $\lim_{t \to 0} \frac{sin\, t}{t} = 1$.

3. Let f be defined in some open interval containing $x = a$. Define $f'(a)$, the derivative of f at $x = a$. Note: There are two standard definitions.

4. Interpret $f'(a)$ in terms of a tangent line and as a rate of change.

5. Exercise: Show that the two limit definitions for the derivative are equivalent.

6. Exercise: Let f be defined in some open interval containing $x = a$. Show that if f is differentiable at $x = a$, then f is continuous at $x = a$.

7. Exercise: Prove the linear approximation theorem.

Linear Approximation Theorem:

Suppose f is defined in a neighborhood of x_0. Then f is differentiable at x_0 if and only if there exists a linear (affine) function

$A(x) = a(x - x_0) + f(x_0)$ that approximates f near x_0 in the sense that

$$\lim_{x \to x_0} \frac{f(x) - A(x)}{|x - x_0|} = 0 .$$

8. Use the linear approximation theorem to generalize the notion of differentiability to functions of several variables.

9. How do you locate the extreme values of a function?

Theorem. If f has a relative maximum at $x = x_0$ and is differentiable at $x = x_0$, then $f'(x_0) = 0$.

10. Mean Value Theorem.

Suppose f is differentiable on (a,b) and continuous on $[a,b]$. Then there exists c in (a,b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

11. How do you determine the shape of the graph of a function?

Theorem. If $f'(x) > 0$ on (a,b) then f is strictly decreasing on (a,b) .

12. State and prove the 1st Derivative Test.

13. State and prove the 2nd Derivative Test

14. If two functions f and g have the same derivative, how are f and g related?

15. The derivatives of a function f are often used to determine the behavior of f . How do they characterize the function f ?