Theorem. *Z*, the set of all integers, is a countably infinite set. ($Z \approx J$)

Proof: Define $f : J \to Z$ by $\begin{cases} f(1) = 0 \\ f(n) = \dfrac{n}{2} \quad \text{if} \quad n \text{ is even} \\ f(n) = -\left(\dfrac{n-1}{2}\right) \quad \text{if} \quad n \text{ is odd}, n > 1 \end{cases}$

We now show that f maps J onto Z. Let $w \in Z$. If $w = 0$, then note that $f(1) = 0$. Suppose $w > 0$. Then $f(2w) = \dfrac{2w}{2} = w$. Suppose $w < 0$. Solving $w = -\left(\dfrac{n-1}{2}\right)$ *for* n, we get $n = -2w + 1$. Note that $-2w + 1$ is an odd positive number. So, $f(-2w+1) = -\left(\dfrac{-2w+1-1}{2}\right) = w$. Hence, f maps J onto *Z.*

The function f is 1-1 since $\dfrac{n}{2} = \dfrac{m}{2}$ *implies* $n = m$ *and* $-\left(\dfrac{n-1}{2}\right) = -\left(\dfrac{m-1}{2}\right)$ *implies* $n = m$.

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