Riemann-Darboux Integral

Let f be a bounded function on $[a,b]$. Let P: $a = x_0 < x_1 < ... < x_n = b$ be a partition of $[a,b]$.

$M(f,[x_{i-1},x_i]) = \sup\{f(x) : x \in [x_{i-1},x_i]\}$

$m(f,[x_{i-1},x_i]) = \inf\{f(x) : x \in [x_{i-1},x_i]\}$

Upper sum of f = $U(f,P) = \sum_{i=1}^{n} M(f,[x_{i-1},x_i])\Delta x_i$

Lower sum of f = $L(f,P) = \sum_{i=1}^{n} m(f,[x_{i-1},x_i])\Delta x_i$

Upper integral of f = $U(f)$ = $\inf\{U(f,P) : P \textit{ is a partition of } [a,b]\}$

Lower integral of f = $L(f)$ = $\sup\{L(f,P) : P \textit{ is a partition of } [a,b]\}$

f is said to be Riemann-Integrable over $[a,b]$ if and only if

$U(f) = L(f)$.

Notation: $\int_a^b f(x)dx$