

Riemann Integral:

Let f be a bounded function on $[a, b]$.

Let $P: a = x_0 < x_1 < \dots < x_n = b$ be a partition of $[a, b]$. The norm of the partition, denoted $|P|$, is equal to $\max\{x_i - x_{i-1} : i = 1, 2, \dots, n\}$.

For $i = 1, 2, \dots, n$ let $c_i \in [x_{i-1}, x_i]$.

$\sum_{i=1}^n f(c_i)(x_i - x_{i-1})$ is called a *Riemann Sum* for f over $[a, b]$.

The function f is said to be *Riemann integrable* on the interval $[a, b]$ provided

$$\lim_{|P| \rightarrow 0} \sum_{i=1}^n f(c_i)(x_i - x_{i-1}) = L .$$

That is, for each $\varepsilon > 0$ there exists $\delta > 0$ such that given any partition

$P: a = x_0 < x_1 < \dots < x_n = b$ with $|P| < \delta$, and given any choice of $c_i \in [x_{i-1}, x_i]$, $i = 1, 2, \dots, n$, it follows that

$$\left| \sum_{i=1}^n f(c_i)(x_i - x_{i-1}) - L \right| < \varepsilon .$$

Notation: The limit is usually denoted by $\int_a^b f(x)dx$.