Math Induction:

Example 1. Show that for each n, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Proof: For each *n* in *J*, let *S(n)* be the statement: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

i) *S(1)* is true since $1 = \sum_{i=1}^{1} i^2 = \frac{1(1+1)(2+1)}{6}$.

ii) In order to apply mathematical induction, we must show that for each *k* in *J*, $S(k) \Rightarrow S(k+1)$. The only way this implication could be false is if there exists some k such that $S(k)$ is true and $S(k+1)$ is false. If $S(k)$ is true, then

$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$. Add $(k+1)^2$ to both sides of this equation and obtain

$$\sum_{i=1}^{k} i^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$\sum_{i=1}^{k+1} i^2 = (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right] = (k+1)\left[\frac{k(2k+1)+6k+6}{6}\right]$$

$$\sum_{i=1}^{k+1} i^2 = (k+1)\left[\frac{2k^2+7k+6}{6}\right]$$

$$\sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

So $S(k+1)$ is true whenever $S(k)$ is true. By the principle of mathematical induction, $S(n)$ is true for each n.