Definition. A sequence $\{a_n\}$ is said to be a Cauchy sequence if for each $\epsilon > 0$, there exists $N > 0$ such that $|a_n - a_m| < \epsilon$ whenever $n, m > N$.

Theorem. Every Cauchy sequence of real numbers is bounded.

Proof.

Let $\{a_n\}$ be a Cauchy sequence. Let $\epsilon = 1776$. There exists $N > 0$ such that $|a_n - a_m| < \epsilon$ whenever $n, m > N$. Let $k > N$. If $n > k$, then $|a_n - a_k| < 1776$. That is, if $n > k$, $a - 1776 < a_n < a_k + 1776$. It also follows that $|a_n| < max\{|a_k + 1776|, |a_k - 1776|\}$ for $n > k$.

Let $B = max\{|a_1|, |a_2|, ..., |a_k|, |a_k + 1776|, |a_k - 1776|\}$. Since $|a_n| \leq B$ for all n , B is a bound for the sequence $\{a_n\}$.

Theorem. A sequence $\{a_n\}$ is a convergent sequence if and only if it is a Cauchy sequence.