

Math 4200

What is a sequence?

Definition.

A sequence of real numbers is simply a function $f : D_m \rightarrow R$, where $m \in Z$ and $D_m = \{n \in Z : n \geq m\}$. Usually, $m = 0$ or $m = 1$. If a sequence is denoted by the letter a , then instead of writing $a(n)$, we write a_n . Notations for the entire sequence include $\{a_n\}$, $\langle a_n \rangle$ and $\{a_n\}_{n=m}^{\infty}$.

Definition. $\lim_{n \rightarrow \infty} a_n = L$ means

for every $\epsilon > 0$, there exists $M > 0$ such that
for each $n \in J$, if $n > M$ then $|a_n - L| < \epsilon$.

Theorem. Let $\{a_n\}$ be a sequence of real numbers. Then $\lim_{n \rightarrow \infty} a_n = L$ if and only if given any open interval (x, y) containing L , (x, y) contains all but finitely many terms of the sequence $\{a_n\}$.