**Math 4200**

Show $\lim_{n\to\infty} \frac{2n+1}{n} = 2$ .

Proof.

1. What needs to be done? Do you remember what it means to write $\lim_{n\to\infty} \mathrm{a}_n = L$?
Intuitively it means that $\mathrm{a}_n$ gets closer and closer to $L$ as $n$ gets larger and larger. What do you mean by "closer and closer". For example, $(2 - \frac{1}{n})$ gets closer and closer to $3$ as $n$ gets larger. Do you want to claim that $\lim_{n\to\infty} (2 - \frac{1}{n}) = 3$ ? Of course not!

2.

$\lim_{n\to\infty} \mathrm{a}_n = L$ means

for every $\epsilon > 0$ , there exists $M > 0$ such that
for each $n \in J$ , if $n > M$ then $|\ \mathrm{a}_n - L\ | < \epsilon$ .

3. You must show this definition is satisfied when $\mathrm{a}_n = \frac{2n+1}{n}$ , and $L = 2$.

4. Let $\epsilon > 0$ . You must now find a positive constant $M$ so that $\left| \frac{2n+1}{n} - 2 \right| < \epsilon$ whenever $n > M$ . How are you going to find such an $M$ ? Will it magically appear to you if you wait long enough? This approach could take forever, another limiting process.

5. Enough thinking about what to do, get to it. You want $\left| \frac{2n+1}{n} - 2 \right| < \epsilon$ ; start with this inequality and see if you can find some large values of $n$ that will work.

$\left| \frac{2n+1}{n} - 2 \right| = \left| \frac{2n+1-2n}{n} \right| = \left| \frac{1}{n} \right| = \frac{1}{n}$ . Now $\frac{1}{n} < \epsilon \iff n > \frac{1}{\epsilon}$ .

By the Archimedean property, there exists $M \in J$ such that $M > \frac{1}{\epsilon}$ and so $\frac{1}{M} < \epsilon$. Suppose $n > M$. Then $\frac{1}{n} < \frac{1}{M} < \epsilon$ and $\left| \frac{2n+1}{n} - 2 \right| = \left| \frac{2n+1-2n}{n} \right| = \left| \frac{1}{n} \right| = \frac{1}{n} < \frac{1}{M} < \epsilon$ .

6. Now write up a "book proof".

Let $\epsilon > 0$ . By the Archimedean property, there exists $M \in J$ such that $M > \frac{1}{\epsilon}$. Then $\frac{1}{M} < \epsilon$. Suppose $n > M$. Then $\left| \frac{2n+1}{n} - 2 \right| = \left| \frac{2n+1-2n}{n} \right| = \left| \frac{1}{n} \right| = \frac{1}{n} < \frac{1}{M} < \epsilon$ . Therefore $\lim_{n\to\infty} \frac{2n+1}{n} = 2$ .