Math 4200

What is meant when we write $\lim_{x \to a} f(x) = L$?

a) Intuitive Definition:

Limit of *f(x)* as *x* approaches *a* equals *L* means that *f(x)* gets closer and closer to *L* as *x* gets nearer and nearer to *a.*

What do you mean by "closer and closer" and "nearer and nearer"?

b) Suppose f is a function defined in a neighborhood of the point $x = a$. ($This\ means\ that\ f\ is\ defined\ in\ some\ open\ interval\ conaining\ x = a,\ except\ perhaps\ at\ the\ point\ x = a.$) Then $\underline{f\ has\ a\ limit\ as\ x\ approaches\ a}$ provided there exists a number L such that

for each $\epsilon > 0$, there exists $\delta > 0$ such that

if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

We write $\lim_{x \to a} f(x) = L$.

c) $\lim_{x \to a} f(x) = L$ provided

for each $\epsilon > 0$, there exists $\delta > 0$ such that if *x* is not equal to *a* and the distance from *x* to *a* is less than δ then the distance from *f(x)* to L is less than ϵ.

d) $\lim_{x \to a} f(x) = L$ provided

for each $\epsilon > 0$, there exists $\delta > 0$ such that
if $x \neq a$ and $a - \delta < x < a + \delta$, then $L - \epsilon < f(x) < L + \epsilon$.

e) How can $\lim_{x \to a} f(x)$ fail to exist?

f) What is meant when we write $\lim_{x \to a^+} f(x) = L$?

What is meant when we write $\lim_{x \to a^-} f(x) = L$?