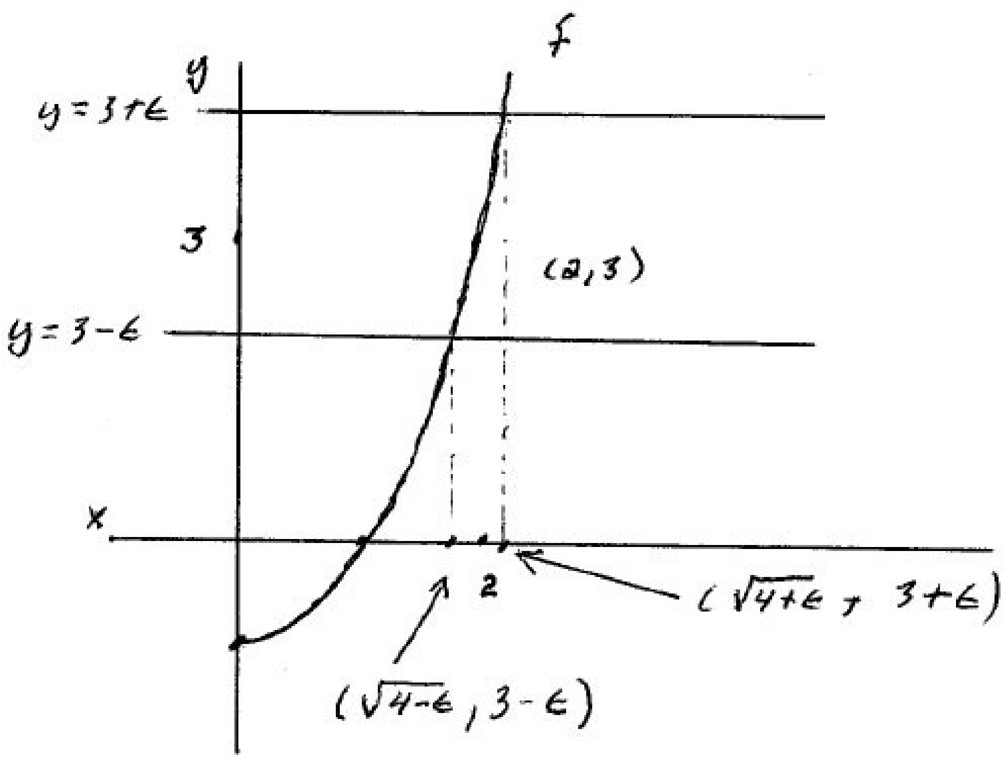
**Math 4200**

Suppose $f(x) = x^2 + 1$. Show that $\lim_{x \to 2} x^2 - 1 = 3$.

Proof:

Let $\varepsilon > 0$. We must find $\delta > 0$ so that $|f(x) - 3| = |x^2 - 1 - 3| < \varepsilon$ whenever $0 < |x - 2| < \delta$. How should we find $\delta$? Consider the graph of $f$.




Let $\delta_1 = 2 - \sqrt{4 - \varepsilon}$ , $\delta_2 = \sqrt{4 + \varepsilon} - 2$. Choose $\delta = \min\{\delta_1, \delta_2\}$.

Suppose $0 < |x - 2| < \delta$. Then $-\delta < x - 2 < \delta$, $-\delta_1 < x - 2 < \delta_2$, $2 - \delta_1 < x < 2 + \delta_2$,

$\sqrt{4 - \varepsilon} < x < \sqrt{4 + \varepsilon}$, $4 - \varepsilon < x^2 < 4 + \varepsilon$, $3 - \varepsilon < x^2 - 1 < 3 + \varepsilon$ ,

$|f(x) - 3| = |x^2 - 1 - 3| < \varepsilon$ .