Math 4200 Assignment

Definition. Suppose f is a function defined in a neighborhood of the point $x = a$. ($This\,means\,that\ f\ is\,defined\,in\,some\,open\,interval\,conaining\ x\ =\ a,\ except\ perhaps\,at\,the\,point\ x = a.$) Then $\underline{f\ has\,a\,limit\,as\ \ x\ approaches\ \ a}$ provided there exists a number L such that

for each $\epsilon > 0$, there exists $\delta > 0$ such that

if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

We write $\lim_{x \to a} f(x) = L$.

Definition. Suppose f is a function defined in some open interval containing $x = a$. Then $\underline{f\ is\,continuous\,at\ x = a}$ provided

$$\lim_{x \to a} f(x) = f(a) \ .$$

The function *f is continuous on an open interval (a,b)* provided f is continuous at each point of (a,b) .

1. Suppose $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$. Prove the following:

a) $\lim_{x \to a} f(x) + g(x) = L + M$

b) $\lim_{x \to a} c \cdot f(x) = c \cdot L \quad (c\ is\,a\,constant)$

c) $\lim_{x \to a} f(x) \cdot g(x) = L \cdot M$

d) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M} \quad (Assume\ M \neq 0\ and\ g(x) \neq 0\ for\,all\ x\ .)$

2. Use the definition of the limit to decide whether the following limits exist at the indicated point.

a) $\lim_{x \to 4} \sqrt{x}$

b) $\lim_{x \to 0} \frac{|x|}{x}$

c) $\lim_{x \to 1} \frac{x^3 - x^2}{x - 1}$

3. Suppose $\lim_{x \to a} f(x) = 0$ and g is a bounded function.

Prove that $\lim_{x \to a} f(x) \cdot g(x) = 0$.

4. State and prove a "squeeze theorem" for limits of functions.

5. Let f be a function which is continuous at $x = a$. Prove that there exist $K > 0$, $\delta > 0$ such that $|f(x)| < K$ for all x in the interval $(a - \delta \quad , a + \delta)$.