

**Theorem.**  $\sqrt{5}$  is an irrational number.

**Proof**

Suppose that  $\sqrt{5}$  is an irrational number. Then  $\sqrt{5} = \frac{a}{b}$  where  $a$  and  $b$  are integers,  $b \neq 0$ . Then  $5 = \frac{a^2}{b^2}$  and  $5b^2 = a^2$ .

Consider both sides of the last equation. The prime factorization of  $a^2$  is obtained by multiplying the prime factorization of  $a$  by itself. So all of the exponents of the primes (after simplifying) in the factorization of  $a^2$  are even. The prime factorization of  $5b^2$  is obtained by multiplying 5 by the prime factorization of  $b^2$ . Since the exponents of the primes in the factorization of  $b^2$  (after simplifying) are all even, the exponent of 5 in the factorization of  $5b^2$  must be odd.

This is a logical contradiction since the two prime factorizations must agree. The supposition that  $\sqrt{5}$  is an irrational number led to this contradiction. Therefore,  $\sqrt{5}$  is an irrational number.