

Urn models are used in physics to model diffusion and other related phenomena. Suppose that an urn contains 15 red tickets, 7 white tickets, and 3 blue tickets. A ticket is chosen from the urn and its color noted. The ticket is returned to the urn along with 5 additional tickets of the same color. The urn is shaken and a second ticket is drawn and its color noted.

- a) What is the chance the first ticket is red?

$$\frac{15}{25}$$

- b) What is the probability that the second ticket is white?

$B_1$ : "red on 1st draw" ,  $B_2$ : "white on 1st draw" ,  $B_3$ : "blue on 1st draw"

$A$ : "second ticket is white"

$$P(A) = P(A | B_1) \cdot P(B_1) + P(A | B_2) \cdot P(B_2) + P(A | B_3) \cdot P(B_3)$$

$$P(A) = \frac{7}{30} \cdot \frac{15}{25} + \frac{12}{30} \cdot \frac{7}{25} + \frac{7}{30} \cdot \frac{3}{25}$$

- c) Given that the second ticket is white, what is the chance the first ticket was white?

$$P(B_2 | A) = \frac{P(B_2 \cap A)}{P(A)} = \frac{\frac{12}{30} \cdot \frac{7}{25}}{\frac{7}{30} \cdot \frac{15}{25} + \frac{12}{30} \cdot \frac{7}{25} + \frac{7}{30} \cdot \frac{3}{25}}$$