PROBABILITY RULES (The World)

Definition: The probability [chance] of event A is the proportion [percentage] of the time A is expected to happen when the random process is repeated over and over again.

Fundamental Counting Principle: If event A can occur in m ways and after A occurs event B can occur in n ways, then the number of ways both events A and B can occur is m x n .

Permutations and Combinations: The number of ways k objects can be selected from n objects and arranged in order is

$$_nP_k = \frac{n!}{(n-k)!}$$

The number of ways k objects can be selected from n objects without regard to order is

$$_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Opposite Event Rule: The probability that event A happens is equal to one minus the probability that A doesn't happen.

$$P(A) = 1 - P(A^c)$$

Multiplication Rule: The probability that events A and B both happen is equal to the probability that A happens times the probability that B happens given that event A has occurred.

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

Conditional Probability Rule:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \quad \text{if} \quad P(B) > 0 .$$

Addition Rule: The probability that event A or event B happens is equal to the probability that A happens plus the probability that B happens minus the probability that both happen.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Definition: Two events are independent if when one happens, the probability that the other happens is unchanged. If events A and B are independent, then

$$P(A \cap B) = P(A) \cdot P(B \mid A) = P(A) \cdot P(B)$$

Law of Total Probability:

Suppose $S = B_1 \cup B_2 \cup \ldots \cup B_n$ and $B_i \cap B_j = \varnothing$ for $i \neq j$.
Then for any event A,

$$P(A) = \sum_{i=1}^{n} P(A \mid B_i) \cdot P(B_i)$$

Bayes' Rule: Suppose $S = B_1 \cup B_2 \cup \ldots \cup B_n$ and $B_i \cap B_j = \varnothing$ for $i \neq j$. And for all i, $P(B_i) > 0$.

Then given any event A ,

$$P(B_j \mid A) = \frac{P(A \mid B_j) \cdot P(B_j)}{\sum_{i=1}^{n} P(A \mid B_i) \cdot P(B_i)}$$