1. Let f be a function defined in an open interval containing the point *x = a.*

A **Taylor series** is a representation of the function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point.

If $f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$ for every x in the interval, then f is said to be

represented by its Taylor series about *x=a* and f is uniquely characterized

by the values $f^{(n)}(a)$ *for* $n=1,2,3,\ldots$

Examples: $e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$ *and* $\sin x=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$

2. Suppose $X : \Omega \rightarrow R$ is a random variable. Can we characterize X by a sequence of values?

The answer is yes for most random variable. The values are

$$E(X) \ , \ E(X^2) \ , \ E(X^3) \ , \dots , \ E(X^n) \ , \dots$$

These values are called the *moments of the random variable* X *.*

Now what is the connection to Taylor series?

Definition: Let the real-valued function be defined by $M_X(t) = E\,[e^{tX}]$

for every t in an open interval containing $x = 0$.

3. We now find the derivatives of $M_X(t) = E[e^{tX}]$ at $x = 0$.

$$M'_X(t) = \frac{d}{dt} E[e^{tX}] = E[\frac{d}{dt} e^{tX}] = E[X e^{tX}]$$

Similarly, we get

$$M''_X(t) = E[X^2 e^{tX}]$$

and in general

$$M_X^{(n)}(t) = E[X^n e^{tX}]$$

For each n , $M_X^{(n)}(0) = E[X^n]$.

4. The moment-generating function $M_X(t) = E\,[e^{tX}]$

is characterized by its derivatives at $x = 0$ (think Taylor series) .

These derivatives are the moments of the random variable X and

they uniquely characterize X .